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The distortion of current and voltage waves on transmission lines

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THE DISTORTION OF CURRENT AND VOLTAGE WAVES ON TRANSMISSION LINES.

BY

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I INTRODUCTION

About four years ago certain large power companies in Minnesota suspected that something was wrong on their lines. Mr. D. A. Dewell who was at that time working for a Master's degree in Electrical Engineering at Iowa State College went up there taking with him a Westinghouse portable oscillograph. Mr. Dewell took a large number of oscillograms while on that trip and it was the sight of those pictures which is responsible for the title of this thesis. The wave shape was far from sinusoidal. It was so far off that one could not help but wonder how electrical equipment designed for use on a sine wave might function under such extreme cases of wave distortion.

The question arose as to the cause of such extreme distortion. It was almost unbelievable that the voltage wave as produced by the alternator could possibly have been so badly distorted. The subject of non-sinusoidal waves of current and voltage on transmission lines appeared to be full of possibilities.

A certain amount of experimental work had been done when it occurred to the writer that a very important phase of this subject was to prove either experimentally or mathematically, that a voltage wave which is only slightly distorted may under certain conditions develop into a very badly distorted wave. This proof the writer succeeded in obtaining. As the work progressed other possibilities presented themselves, some of which are considered to be of very great importance.

Due to the large amount of mathematical work involved it was thought best to reserve the first part of this thesis for a discussion of the problem, theory and results. All mathematical work will therefore be found in the appendix which is thoroughly indexed in order that all material appearing in the body of the report may be found with the least possible effort.

It has been gratifying and sometimes alarming to the writer to note the interest which has begun to be manifested in the subject of non-sinusoidal wave shapes since the beginning of this research two years ago. The General Electric Company has placed a harmonic generator on the market. This consists of five machines direct connected with movable stators and is made expressly for colleges and others interested in learning more about the effects of non-harmonic waves. Even power companies are beginning to realize the importance of the subject, as shown by the paper presented at the Kansas City divisional meeting of the American Institute of Electrical Engineers in October 1931 entitled, "Effect of Wave Form on Operation of Induction Type Protective Relays." Here was a subject the writer had already considered and rejected because of the large supply of equipment required.

It is of interest to quote the April 1933 issue of Electrical Engineering on the subject of reactive power.

"The quadrature component accompanying energy flow in inductive and capacitive circuits has vagaries, which relatively speaking have been overlooked. ....To Prof. Constantin D. Busila

of the Polytechnic School at Bucharest Roumania, is given most of the credit for bringing the loose status of flux energy to the fore." "In 1931 the only answer that could even partially be agreed on was that:
....prevailing methods of measuring reactive components are acceptable as a practical expediency, although it is recognized that errors of measurement are incurred under unbalanced and non-sinusoidal conditions. However, the relative unimportance, from the economic standpoint, of reactive power flow as compared with demand and energy elements of electricity costs tends to discount the value of an exhaustive and abstract analysis of the inconsistencies of reactive concepts and the corresponding technique of measurement. In brief, American practise is content with the definition of reactive component as that quantity which is measured when the potential is shifted to quadrature with its appropriate vector position for true power measurement."

The foregoing statements have been quoted to show the "right about face" that has taken place during the last two years and to partially justify the title of this thesis. It is the writer's belief that the contents will make that justification complete.

II LONG DISTANCE TRANSMISSION LINES

A transmission line is said to be "electrically long" if the component of current due to capacity susceptance or to the conductance of the line is a considerable fraction of the rated current of the line. One line may be physically shorter but electrically longer than another line.

When an electric impulse is produced in a conductor, the impulse travels at approximately the speed of light, (for overhead lines; the velocity is considerably less in cables) which has been determined to be $3x10^{10}$ centimeters per second. If the line is open circuited the impulse travels to the far end of the line and is reflected, returning at the same speed. If at the moment of return another impulse is sent into the line it is added to the first impulse and travels out to the end of the line, is reflected and returns to be added to a third impulse and so on. In this way large voltages and currents may be produced. Suppose a 60 cycle sine wave is applied to a transmission line whose length is 776 miles. Since the rate of propagation is $3x10^{8}$ meters per second it becomes evident that one wave length will be $5x10^{6}$ meters or 3106 miles. The time of one complete cycle is the time required for light to travel four times the length of the line. Since the line has a length equal to one quarter wave length, the reflected wave will arrive at the starting point at exactly the instant when another wave is starting out. The result is very similar to the resonant condition of an ordinary series circuit, the only condition limiting the flow of current being the resistance of the line. This condition will hereafter be referred to as the condition of quarter resonance. Since the wave length corresponding to a frequency of 60 cycles per second is 3106 miles, one quarter wave length would be 776 miles. No power line has ever been built which will anywhere nearly approach this length. Hence it would appear that the condition of quarter wave length resonance is one which would not need consideration. If all voltage waves were purely sinusoidal there would be no cause for worry.

However, suppose one of the higher harmonics to be present in the voltage wave of the alternator. Quarter wave resonance will now develop (for the harmonic) on a 258 mile line in the case of a 3rd harmonic, and on a 155 mile line in the case of a 5th. A 7th harmonic requires a line only 109 miles in length to produce quarter wave resonance while an 11th will produce the resonance condition on a line which is only about 70 miles in length. It is therefore evident that if some of the higher harmonics are present a line as short as seventy miles may be electrically long. The problem now develops--how serious is this matter of quarter wave resonance? Does the shape of the voltage wave of the generator have any particular influence on the operation of the line and the equipment connected to it? How large a harmonic in per cent of the fundamental can be used without seriously affecting the economical operation ofthe line? These are some of the questions which must be answered.

III THE DETERMINATION OF OPERATING CONDITIONS ON A 250 MILE TRANSMISSION LINE

1. General Methods.

In solving any transmission line problem we have to deal with the constants of the line, inductance, resistance, conductance and susceptance. In the short lines the capacity of the circuit may usually be neglected. The leakage current may be neglected in practically all power transmission line problems. In such cases the problem becomes very simple and is merely a matter of adding the impedance drop at the proper angle to the load voltage to obtain the generator voltage. As the lines become longer the distributed capacity of the circuit may no longer be neglected. We have at our disposal various approximate methods of handling this type of problem. All of these make use of the same artifice, namely that of considering the capacity, which in reality is distributed, as being concentrated at certain parts of the line. These approximate methods are sufficiently accurate for practical use providing the lines are not too long, the capacity not too great, or the frequency too high.

For the solution of problems involving long lines (sine wave applied) the hyperbolic method gives very accurate results and the work connected with it is not much greater than that of using the π method, the nominal T or the Steinmetz approximation. Even the hyperbolic method as ordinarily used is not accurate on circuits where the higher harmonics are present. Since the parameters of the circuit are constant for the steady state condition, the usual hyperbolic method can be modified to give accurate results by the superposition of separate solutions for the various frequencies.

Although it would not be impossible to develop a set of general equations to represent the case of a long line on which harmonics exist, the equations themselves would be of little value. It would be impossible to look at the equations and get any idea of how the line would behave. It was thought that the greatest practical benefit would result if it could be shown that a wave containing moderate harmonics might become very badly distorted on a line similar to some which are at present in existence. A 250 mile, 60 cycle, 220,000 volt, 75,000 K.W., three phase transmission line was chosen very similar to a line in Southern California. The size of wire, spacing of conductors etc., were chosen according to present standards of good practise. Values of the parameters were taken from tables.

2. Sine Wave Applied.

All calculations were first made for a voltage wave which was purely sinusoidal. (See Appendix for these calculations) The results were grouped together in Table I.

3. Non-harmonic Wave Applied.

The question then arose---how would this same line with the same load (R and L same as before) respond to a voltage which has the same effective value as it leaves the generator, but which contains various harmonics. The calculations were carried through for a wave containing the odd harmonics from the third to the eleventh inclusive. The third harmonic content of this wave was seven per cent of the fundamental. The exact equation of the wave as used appears under heading (5) in the Appendix. The load current was kept the same as before. The results of these calculations have been tabulated in Tables II and III.

Table III was calculated for a non-harmonic wave. The full load calculation was made first. Since the generator voltage was 138,700 volts in the case of a sine wave at full load the non-harmonic wave was so chosen as to give a generator voltage of 138,700 volts at full load. The equation of this wave will be found in the Appendix Part 5. The generator current calculation will be found in Part 10 of the Appendix. The effective value of the load voltage was calculated in Part 8 of the Appendix. The load current of 198 amperes was chosen at this value because that was full load for the sine wave. In other words the full load was 198 amperes and the generator voltage had to be 138,700 volts. The magnitude of the harmonics was so chosen that this condition existed. The K.V.A. at the load was calculated in accordance with the A.I.E.E. definition of apparent power and is simply the product of the r.m.s. values of current and voltage at the load. The power factor at the load was also calculated in accordance with the A.I.E.E. definition.

In making the no load calculation the same procedure was followed with a slight modification. From Table I for the sine wave it was found that the voltage at the generator was 111,000 volts at no load. The percentage of the harmonics was reduced until the effective value became 111,000 volts. The current calculation was made in Part 11 of the Appendix. The load voltage was found by substituting in the voltage equation in Part 11 and finding the effective value of all the load voltages and taking the square root of the sum of their squares.

4. Results and Discussion.

It should be noted that under full load conditions the 3rd harmonic of e.m.f. at the load end of the line is 29% of the fundamental. At this time it is well to remember that the 3rd harmonic of e.m.f. at the generator was only seven per cent of the fundamental. When the load is removed the third harmonic of e.m.f. becomes about 53% of the fundamental. Obviously, when conditions such as these can exist it is important to know how various kinds of apparatus, motors, power factor meters, relays, etc., will function on waves which are not sinusoidal.

TABLE I

Operating Characteristics of a 250 Mile Transmission Line. Sine
Constant Load Voltage. $E_0 = 127,020$ volts (line to neutral)

I_0	E_{gen}	I_{gen}	E_{gen}	I_{gen}	K.V.A.gen per phase
Unity Power Factor at Load					
0	111,000 + j2,930	-1.6 + j164	111,000	164	18,200
40	112,330 + j10,360	33.4 + j164.9	112,600	168.3	18,930
80	113,660 + j17,470	68.4 + j165.8	115,000	179	20,600
120	115,000 + j24,930	103.4 + j166.7	117,600	196.4	23,100
160	116,330 + j32,230	138.4 + j167.6	120,800	217.5	26,200
197	117,560 + j38,930	170.4 + j168.5	123,800	239	29,600
Power Factor at Load-.85 Lagging					
0	111,000 + j2,930	-1.6 + j164	111,000	164	18,200
40	116,000 + j8,450	28.6 + j146.4	116,200	149	17,300
80	121,000 + j13,970	58.7 + j128.8	121,500	141.5	17,200
120	126,000 + j19,490	89.1 + j111	127,300	142	18,100
160	131,000 + j25,010	119.4 + j93.5	133,700	151.6	20,230
197	135,600 + j30,130	147.4 + j77.3	138,700	166.5	23,100

E and I with reference to E_0

TABLE I

tics of a 250 Mile Transmission Line. Sine Wave Applied.
. E_0 = 127,020 volts (line to neutral)

I_{gen}	E_{gen}	I_{gen}	K.V.A.gen per phase	K.W.gen per phase	K.W.load per phase	%Eff.	P.F.g
	Unity Power Factor at Load						
-1.6 + j164	111,000	164	18,200	303.5	0	0	.016
33.4 + j164.9	112,600	168.3	18,930	5460	5100	93.5	.288
68.4 + j165.8	115,000	179	20,600	10,670	10,170	95.2	.518
103.4 + j166.7	117,600	196.4	23,100	16,060	15,260	95	.695
138.4 + j167.6	120,800	217.5	26,200	21,540	20,350	94.6	.822
170.4 + j168.5	123,800	239	29,600	26,560	25,000	94.3	.898
	Power Factor at Load-.85 Lagging						
-1.6 + j164	111,000	164	18,200	303.5	0	0	.016
28.6 + j146.4	116,200	149	17,300	4,560	4,330	95.2	.26
58.7 + j128.8	121,500	141.5	17,200	8,900	8,650	97.3	.517
89.1 + j111	127,300	142	18,100	13,390	13,000	97	.74
119.4 + j93.5	133,700	151.6	20,230	17,990	17,300	96.2	.887
147.4 + j77.3	138,700	166.5	23,100	22,200	21,300	95.9	.96

to E_0

TABLE II

This table gives the effective values of the harmonics of current and voltage and their phase relations. It also gives the power contributed by each of the harmonics. The table is based on a non-harmonic generator voltage of 138,700 volts, (effective value) and a load whose resistance and inductance is the same as used in the case of a sine wave to give .85 power factor. The effective value of the load current is 198 amperes.

Harmonic	E_{gen}	I_{gen}	P_{gen} (in K.W.)
1	138,000 + j0	159.1800 + j43.53	21,970.00
3	10,600 + j0	65.8400 + j62.88	698.00
5	6,660 + j0	1.1230 - j16.35	6.35
7	4,240 + j0	1.0965 - j07.02	4.65
9	2,120 + j0	-11.0000 - j35.67	-23.30
11	495 + j0	-.0060 + j01.35	-2.97

Total power at generator = 22,652 K. W.

Each harmonic of current is expressed with respect to its own harmonic of voltage. These data are used for Table III. See Appendix Part 10 for mathematical work connected with the determination of this table.

TABLE III

This table gives the same quantities given in Table I for the same transmission line, except that the original voltage wave is not sinusoidal but has the equation given in Part 5 of the Appendix. The voltages and currents given in this table are effective values.

I_{gen}	E_{gen}	E_{gen}	I_{gen}	K.V.A. load	K.W. load per phase	K.W. gen per phs	% Eff.	P.F. load	P.F. gen
202	111,000	140,000	0	0	0	634	0	---	.0283
192.5	138,700	132,600	198	26,250	21,502	22,652	94.9	.82	.848

For discussion of this table see page

TABLE IV

*This table gives the value of each of the harmonics and the resultant voltage for each ten degrees along the fundamental scale. This is for the voltage wave as it leaves the generator.

θ	e_1	e_3	e_5	e_7	e_9	e_{11}	e_r
0	0	-4,460	-2,742	-1,044	1,726	297	-6,223
10	34,000	3,330	-7,530	-5,200	-2,460	-697	21,443
20	67,000	10,230	-6,930	4,620	-1,726	181	73,375
30	97,500	14,370	-1,394	-2,060	2,460	575	111,451
40	125,700	14,650	5,160	-6,000	1,726	-575	140,661
50	149,800	11,020	8,000	-2,060	-2,460	-181	164,119
60	169,000	4,440	5,160	4,620	-1,726	697	182,191
70	183,600	-3,330	-1,394	5,200	2,460	-297	186,239
80	192,500	-10,230	-6,930	-1,044	1,726	-497	175,525
90	195,000	-14,370	-7,530	-5,900	-2,460	635	165,375
100	192,500	-14,650	-2,740	-3,000	-1,726	61	170,425
110	183,600	-11,020	4,000	3,870	2,460	-637	182,273
120	169,000	-4,440	7,900	5,630	1,726	403	180,219
130	149,800	3,330	6,130	0	-2,460	637	157,437
140	125,700	10,230	0	-5,630	-1,726	-678	127,896
150	97,500	14,370	-6,130	-3,870	2,460	61	104,391
160	67,000	14,900	-7,900	3,000	1,726	297	79,023
170	34,000	11,020	-4,000	5,930	-2,460	-495	43,995
180	0	4,460	2,740	1,044	-1,726	-297	6,221
190	-34,000	-3,333	7,530	-5,200	2,460	697	-31,843
200	-67,000	-10,230	6,930	-4,600	1,726	-181	-73,335
210	-97,500	-14,370	1,394	2,060	-2,460	-575	-111,451

*See equation under Heading No.5 in the appendix.

TABLE IV Continued

220	-125,700	-14,650	-5,160	6,000	-1,726	575	-140,661
230	-149,800	-11,020	-8,000	2,060	2,460	181	-164,119
240	-169,000	-4,440	-5,160	-4,620	1,726	-697	-182,191
250	-183,600	3,330	1,394	-5,200	-2,460	297	-186,239
260	-192,500	10,230	6,930	1,044	-1,726	497	-175,525
270	-195,000	14,370	7,530	5,900	2,460	-635	-165,375
280	-192,500	14,650	2,740	3,000	1,726	-61	-170,425
290	-183,600	11,020	-4,000	-3,870	-2,460	637	-182,273
300	-169,000	4,440	-7,900	-5,630	-1,726	-403	-180,019
310	-149,800	-3,330	-6,130	0	2,460	-637	-157,437
320	-125,700	-10,230	0	5,630	1,726	678	-127,896
330	-97,500	-14,370	6,130	3,870	-2,460	-61	-104,391
340	-67,000	-14,900	7,900	-3,000	-1,726	-297	-79,023
350	-34,000	-11,020	4,000	-5,930	2,460	495	-43,995
360	0	-4,460	-2,740	-1,044	1,726	297	-6,221

TABLE V

*Data for plotting voltage wave at the load under open circuit conditions.

θ	e_1	e_3	e_5	e_7	e_9	e_{11}	e_r
0	-5,870	0	3,750	1,260	-15,930	401	-16,389
10	33,100	-51,700	9,500	-5,630	11,900	-1,068	-3,898
20	71,000	-89,800	8,450	-6,120	15,930	332	792
30	107,000	-103,400	1,390	2,140	-11,900	845	-3,925
40	140,000	-89,800	-6,700	6,563	-15,930	-910	33,223
50	167,000	-51,700	-9,940	2,360	11,900	-223	119,597
60	191,000	0	-6,130	-4,970	15,930	149	195,979
70	208,500	51,700	2,080	-5,700	-11,900	-503	244,177
80	219,000	89,800	8,800	1,032	-15,930	-718	318,632
90	223,500	103,400	9,260	6,480	11,900	993	355,533
100	221,000	89,800	3,080	3,410	15,900	37	333,257
110	213,000	51,700	-5,270	-4,140	-11,900	-1,020	242,370
120	196,500	0	-9,800	-6,230	-15,930	730	165,270
130	175,000	-51,700	-7,420	-115	11,900	567	128,232
140	148,000	-89,800	348	-6,150	15,930	-1,045	79,583
150	117,300	-103,400	7,880	4,330	-11,900	149	14,359
160	82,000	-89,800	9,760	-3,200	-15,930	945	-16,225
170	44,600	-51,700	4,680	-6,480	11,900	-795	2,205
180	5,870	0	-3,750	-1,260	15,930	-401	16,389
190	-33,100	51,700	-9,500	5,630	-11,900	1,068	3,898
200	-71,100	89,800	-8,450	5,120	-15,930	-332	-792
210	-107,000	103,400	-1,380	-2,140	11,900	-845	3,935

*See equation under Heading No.7 in the appendix.

TABLE V Continued

	1	3	5	7	9	11	r
220	-140,000	89,800	6,650	-6,563	15,930	910	-33,273
230	-167,200	51,700	9,945	-2,360	-11,900	223	-119,592
240	-191,000	0	6,120	4,970	-15,930	-149	-195,989
250	-208,500	-51,700	-2,070	5,700	11,900	503	-244,167
260	-219,000	-89,800	-8,770	-1,032	15,930	718	-318,602
270	-223,500	-103,400	-9,230	-6,480	-11,900	-993	-355,503
280	-221,000	-89,800	-3,080	-3,410	-15,930	-37	-333,257
290	-213,000	-51,700	5,270	4,140	11,900	1,020	-242,370
300	-196,500	0	9,880	6,230	15,930	-730	-165,270
310	-175,000	51,700	7,420	115	-11,900	-567	-128,232
320	-148,000	89,800	-348	-6,150	-15,930	1,045	-79,583
330	-117,300	103,400	-7,880	-4,440	11,900	-149	-14,359
340	-82,000	89,800	-9,760	3,200	15,930	-945	16,225
350	-44,600	51,700	-4,680	6,480	-11,900	795	2,205
360	-5,870	0	3,750	1,260	-15,930	401	-16,389

TABLE VI

*This table gives the value of each of the harmonics and the resultant current for each ten degrees along the fundamental scale. This is the current wave at the load.

θ	i_1	i_3	i_5	i_7	i_9	i_{11}	i_r
0	-192.6	-36.6	-4.66	2.84	-4.60	.246	-235.62
10	-155.5	-45.8	.46	.83	-4.36	.025	-204.34
20	-110.7	-42.7	5.26	-2.28	4.60	-.229	-146.05
30	- 68.2	-28.1	6.28	-2.38	4.37	.200	- 87.83
40	- 20.7	- 5.9	2.84	.65	-4.60	.104	- 27.59
50	27.4	17.9	-2.65	2.83	-4.37	-.253	40.86
60	74.6	36.6	-6.23	1.29	4.60	.068	110.93
70	119.5	45.8	-5.37	-1.95	4.37	.206	162.56
80	161.0	42.7	- .66	- .65	-4.60	-.210	197.58
90	198.0	28.1	4.52	.15	-4.37	-.063	226.34
100	228.0	5.9	6.47	2.72	4.60	.025	247.69
110	251.3	-17.9	3.80	1.71	4.37	-.110	243.17
120	269.0	-36.6	-1.57	-1.55	-4.60	-.178	224.49
130	276.0	-45.8	-5.83	-2.77	-4.37	.231	217.46
140	274.0	-42.7	-5.91	- .34	4.60	.019	229.70
150	266.0	-28.1	-1.78	2.54	4.37	-.229	242.90
160	249.0	- 5.9	3.63	2.08	-4.60	.148	244.38
170	221.5	17.9	6.45	-1.12	-4.37	.142	240.50
180	192.8	36.6	4.66	-2.84	4.60	-.246	235.57
190	155.4	45.8	- .46	- .83	4.37	.025	205.47
200	115.0	42.7	-5.25	2.28	-4.60	.025	205.47

*See equation under Heading No.9 in the appendix.

TABLE VI Continued

210	68.3	28.1	-6.27	2.38	-4.37	-.200	87.94
220	20.7	5.9	-3.23	- .65	4.60	-.104	27.20
230	-27.8	-17.9	2.65	-2.83	4.37	.253	- 40.86
240	- 74.7	-36.6	6.23	-1.29	-4.60	-.068	-111.03
250	-119.5	-45.8	5.37	1.94	-4.37	-.206	-162.56
260	-161.0	-42.7	.66	.65	4.60	.210	-197.58
270	-198.0	-28.1	-4.52	- .15	4.37	.063	-226.34
280	-228.0	- 5.9	-6.47	-2.72	-4.60	-.025	-247.70
290	-251.3	17.9	-3.80	-1.71	-4.37	.110	-243.17
300	-269.0	36.6	1.58	1.55	4.60	.178	-224.49
310	-276.0	45.8	5.83	2.77	4.37	-.231	-217.00
320	-274.0	42.7	5.91	.34	-4.60	-.019	-229.67
330	-266.0	28.1	1.78	-2.64	-4.37	.229	-242.80
340	-249.0	5.9	-3.63	-2.08	4.60	-.148	-244.38
350	-221.5	-17.9	-6.45	1.12	4.37	-.143	-240.50
360	-192.8	-36.6	-4.66	2.84	-4.60	.246	-235.57

TABLE VII

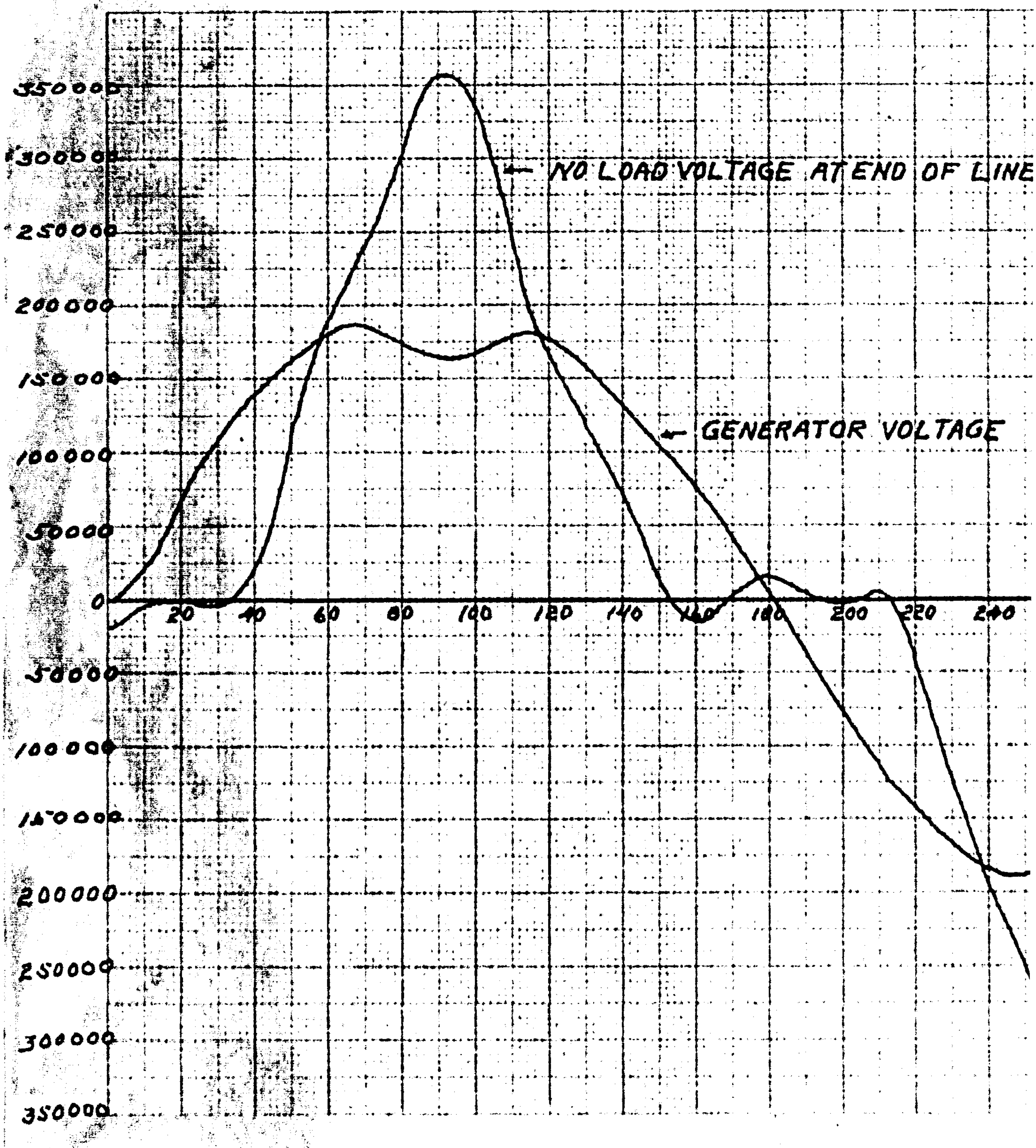
*This table gives the value of each of the harmonics of voltage and the resultant, at the receiving end of line under full load.

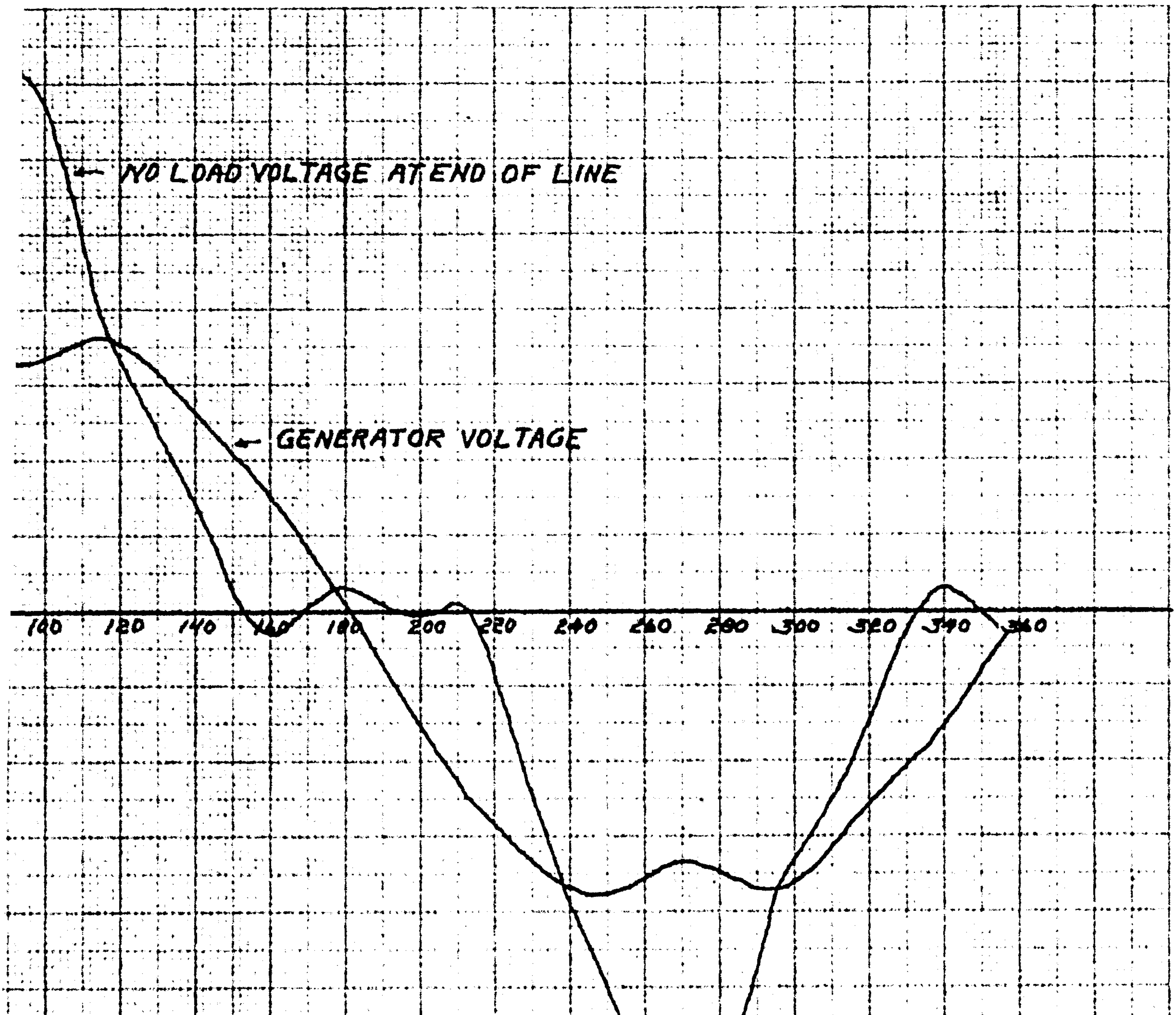
θ	e_1	e_3	e_5	e_7	e_9	e_{11}	e_r
0	-38,000	-47,700	5,030	1,206	-15,870	370	-95,394
10	-7,150	-30,600	11,150	-6,020	11,660	-950	-11,910
20	24,000	-5,370	9,300	-5,330	15,950	282	38,812
30	54,300	21,300	497	2,380	-11,660	760	67,586
40	83,000	42,300	-8,300	6,950	-15,800	-802	107,248
50	109,000	52,000	-11,440	2,380	11,650	-211	163,379
60	132,000	47,700	-6,430	-5,330	15,900	945	184,785
70	151,000	30,600	3,170	-6,020	-11,650	-435	166,665
80	165,000	5,370	10,520	1,206	-15,900	-648	165,548
90	174,500	-21,300	10,320	6,850	11,650	875	182,895
100	178,500	-42,300	2,780	3,475	15,900	43	158,398
110	176,600	-52,000	-6,750	-4,470	-11,650	910	102,640
120	170,000	-47,700	-11,480	-6,520	-15,900	576	88,976
130	158,000	-30,600	-7,950	0	11,650	506	131,606
140	141,000	-5,370	1,202	6,520	15,900	-930	158,322
150	120,200	21,300	9,550	4,470	-11,650	116	143,986
160	95,200	42,300	11,020	-3,475	-15,900	847	129,992
170	67,800	52,000	4,770	-6,850	11,650	-700	128,390
180	38,100	47,700	-5,030	-1,206	15,900	-370	95,094
190	7,150	30,600	-11,140	6,020	-11,650	950	95,580
200	-24,000	5,370	-9,330	5,330	-15,900	-282	-38,812

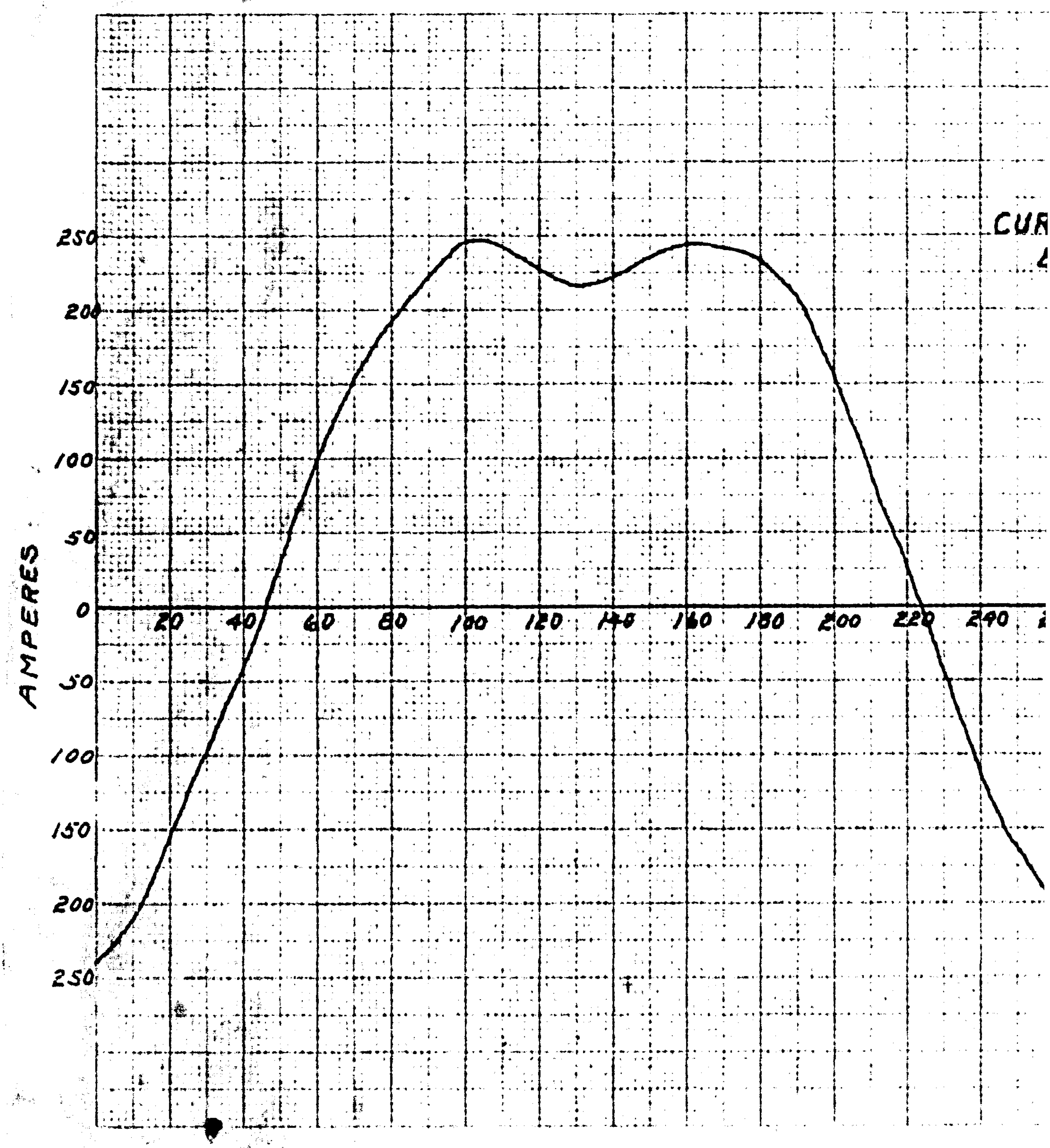
*See voltage equation under Heading No.9 in the appendix.

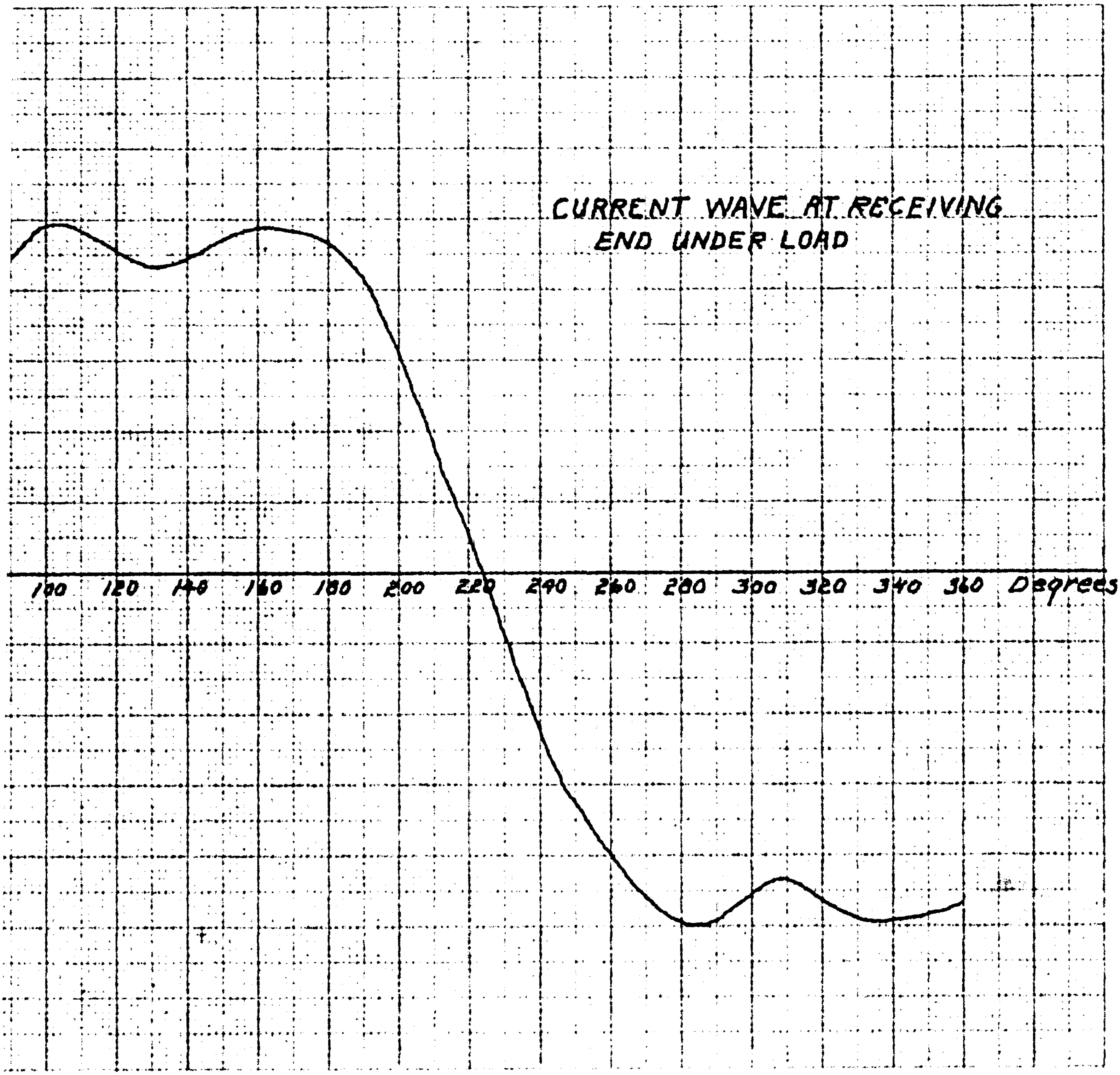
TABLE VII Continued

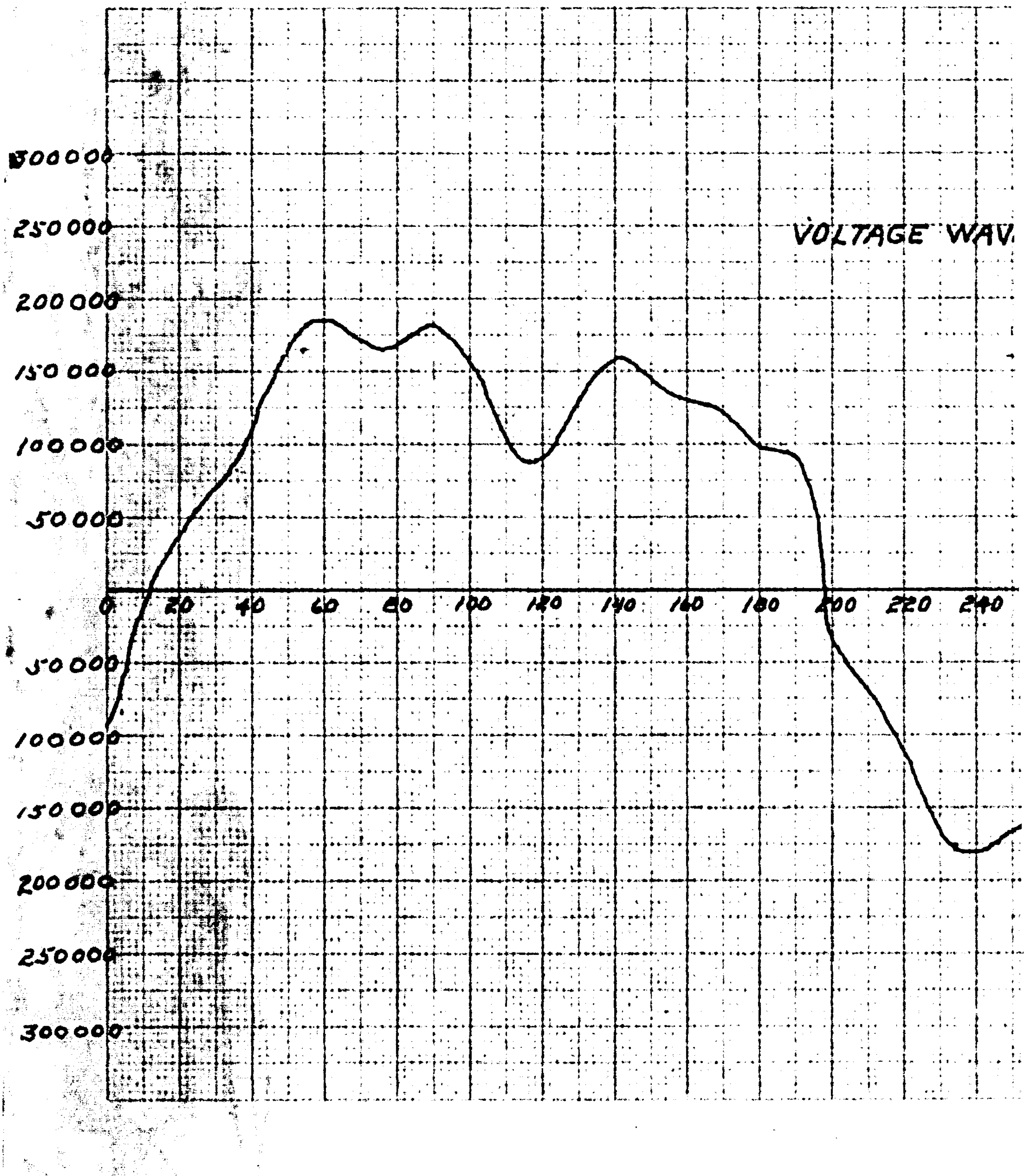
	1	3	5	7	9	11	r
210	-54,300	-21,300	-496	-2,380	11,650	-760	67,586
220	- 83,000	-42,300	8,300	-6,950	15,900	802	107,248
230	-109,000	-52,000	11,440	-2,380	-11,650	211	163,379
240	-132,000	-47,700	6,430	5,330	-15,900	-945	184,785
250	-151,000	-30,600	-3,170	6,020	11,650	435	166,665
260	-165,000	-5,370	-10,520	-1,206	15,900	648	165,548
270	-174,500	21,300	-10,320	-6,860	-11,650	-875	182,895
280	-178,500	42,300	-2,780	-3,475	-15,900	-43	158,398
290	-176,600	52,000	6,750	4,470	11,650	-910	102,640
300	-170,000	47,700	11,480	6,520	15,900	-576	88,976
310	-158,000	30,600	7,950	0	-11,650	-506	131,606
320	-141,000	5,370	-1,202	-6,520	-15,900	930	158,322
330	-120,200	-21,300	-9,550	-4,470	11,650	-116	143,986
340	-95,200	-42,300	-11,020	3,475	15,900	-847	129,992
350	-67,800	-52,000	-4,770	6,830	-11,650	700	128,390
360	-38,100	-47,700	5,030	1,206	-15,900	370	95,094

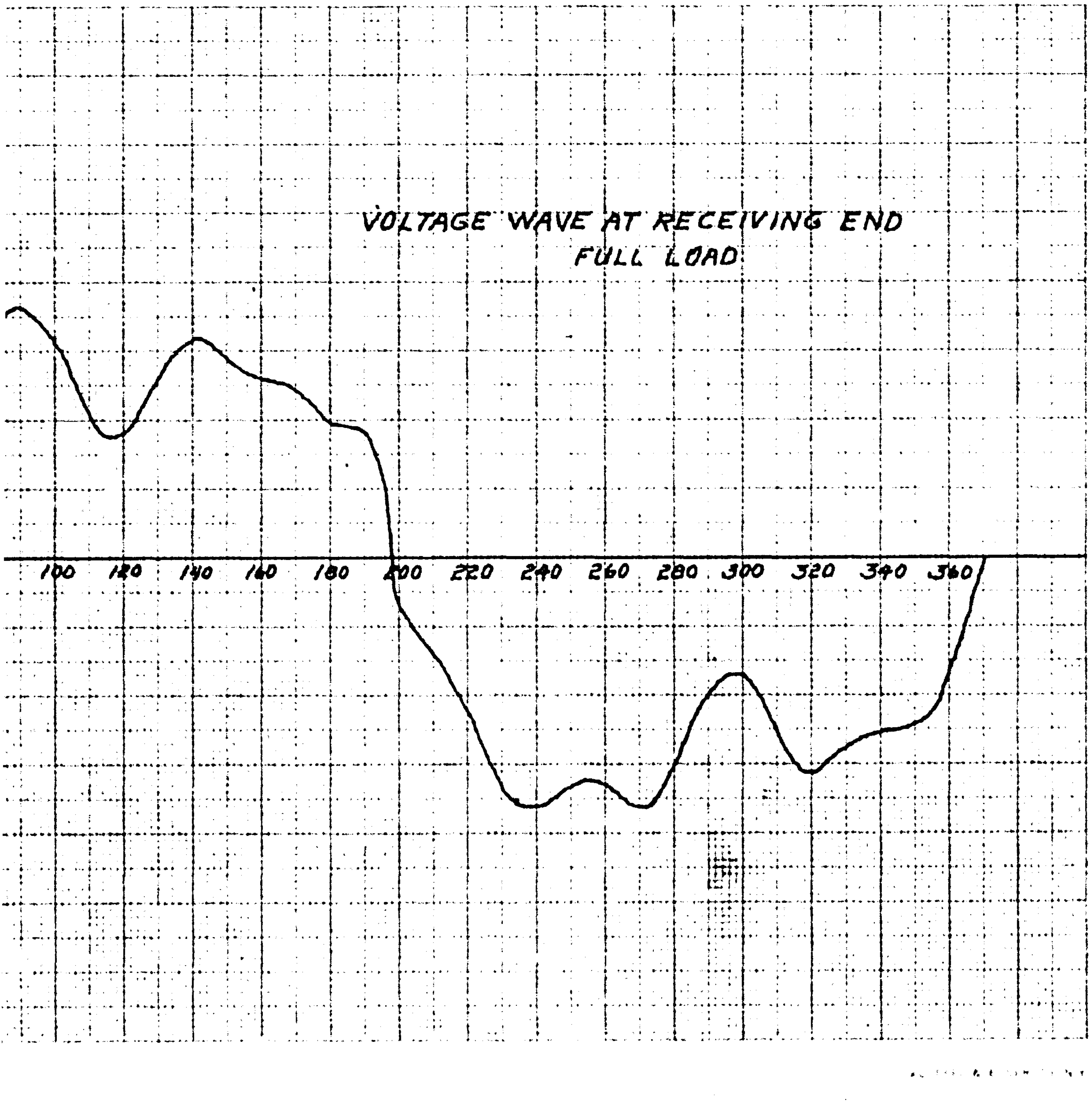

350000
300000
250000
200000
150000
100000
50000
0
50000
100000
150000
200000
250000
300000
350000
20
40
60
80
100
120
140
160
180
200
220
240
NO LOAD VOLTAGE AT END OF LINE
GENERATOR VOLTAGE


NO LOAD VOLTAGE AT END OF LINE
GENERATOR VOLTAGE
100
120
140
160
180
200
220
240
260
280
300
320
340
360


AMPERES
250
200
150
100
50
0
50
100
150
200
250
20
40
60
80
100
120
140
160
180
200
220
240
CUR


CURRENT WAVE AT RECEIVING
END UNDER LOAD
100
120
140
160
180
200
220
240
260
280
300
320
340
360
Degrees


VOLTAGE WAV
300000
250000
200000
150000
100000
50000
0
20
40
60
80
100
120
140
160
180
200
220
240
50000
100000
150000
200000
250000
300000


VOLTAGE WAVE AT RECEIVING END
FULL LOAD
100
120
140
160
180
200
220
240
260
280
300
320
340
360

IV THE DETERMINATION OF POWER FACTOR

In considering the general problem of harmonics on transmission lines it occurred to the writer that the methods being used to measure power factor would probably be in error. The amount of the error would of course depend upon the method used. Letters were therefore written to a number of power companies asking what method they use to determine power factor for large users of power and further, what penalties their power contracts carried for low power factors. If it could be shown that there is a small error in the power factor measurement it might mean that the power companies were getting less returns than they were lawfully entitled to. A survey of the letters indicated that all of the companies written to were using a reactive K.V.A.H. meter and a K.W.H. meter to determine the power factor. In the words of the superintendant of the meter department of one company:

"In all cases where power factor is measured, except for special tests, it is done by use of a K.W.H. meter and a R.K.V.A.H. meter using an auto-transformer to shift the R.K.V.A.H. meter potential to the ninety degree position. The average power factor is then figured from the reading of the two meters at the beginning and end of the billing period. The quotient of the difference of the meter readings gives the tangent of the power factor angle and from the tangent, the cosine is derived".

At this time it may be well to quote the A.I.E.E. definition of power factor. It is as follows;

"The power factor is the ratio of the power to the apparent power." Quoting again: "The apparent power or volt-amperes in an alternating current circuit is the product of the r.m.s. value of the voltage across the circuit by the r.m.s.

value of the current in the circuit. Apparent power is also expressed in kilovolt-amperes, abbreviated kva."

It therefore appears that the true power factor (according to definition) is obtained by dividing power by apparent power. It is obvious that when dealing with sine waves the method outlined above, of dividing the R.K.V.A.H. meter reading by the K.W.H. meter reading to obtain the tangent of the power factor angle gives the true power factor. However, this does not hold true for non-sinusoidal waves. See Appendix for calculation of power, apparent power, power factor, and reactive factor for current and voltage waves which are non-sinusoidal.

It should be noted that the power factor as found by the two methods, for the 250 mile line considered, differs by 7 per cent. It should also be noted that all components of power and reactive power in this case were positive. This is not always the case and would not be the case any time when there are harmonics of voltage and current which are displaced from one another by more than ninety degrees.

It will now be of interest to investigate upon an economic basis, the possible effect of a 7% error in determining the power factor. Quoting a contract used by a large power company:

"Consumer agrees that the following changes in the energy portion of the rate specified in this section herein shall be effective for variations in the average power factor during any month of this agreement;--

<u>If the power factor is</u>

100% to 95%	3% rate decrease
94% to 85%	2% " "
84% to 80%	0% " "
79% to 75%	2% " increase
74% to 70%	3% " "
69% to 65%	4% " "
64% to 60%	6% " "
59% to 50%	8% " "

If at any time the power factor at the points of delivery shall become less than 50% the Power Company may at its option suspend delivery of energy hereunder until power factor has been increased above 50 per cent."

The following is a quotation from this same company's letter concerning the clause just given:

"The above clause is in effect with a customer using 2,000,000 to 2,500,000 K.W.H. per month at maximum load of 6,000 to 7,000 K.W."

In order to get an idea of the penalty inflicted upon the customer of paying on the basis of .75 power factor instead of .82 we will quote rates from another large electric company entitled,

"Unlimited Service Wholesale Primary Energy Rate."

<u>Rate:</u>

<u>Demand Charge:</u>

\$250.00 per month for the first 100 K.W. of maximum demand.
\$2.00 per kilowatt per month for the next 300 K.W. of maximum demand.
\$1.25 per kilowatt per month for all demand in excess of 400 K.W.

<u>Energy Charge:</u>

1.1 ¢ per K.W.H. for first 100 hours use of Customer's maximum demand.
1.0 ¢ per K.W.H. for the next 100 hours use of the Customer's maximum demand.
0.9 ¢ per K.W.H. for the next 100 hours use of Customer's

maximum demand.
0.8 ¢ per K.W.H. for the next 100 hours use of the Customer's maximum demand."

Making the calculation for a customer using 2,500,000 K.W.H. per month we find the monthly bill to be \$33,300.00 or \$399,600.00 per year. If the power factor had been recorded .82 there would be no rate increase or decrease. However, since the power factor is .75 the rate is increased 2 per cent. In other words the customer is paying about \$8000.00 per year (the interest at 4% on a \$200,000.00 bond) penalty for a power factor which according to meter readings is low but according to definition is high enough to require no rate increase.

Present design of lines would call for much larger conductors than would ordinarily be used due to the extremely low cost of copper at this time. Hence low I^2R losses will be attained, but if harmonics are present and quarter wave resonance occurs the currents and voltages will be just that much greater and might be dangerously high.

V METER EXPERIMENTS

The writer was interested in the accuracy of watt-hour and reactive-kilovolt-ampere-hour meters on non-sinusoidal waves chiefly from the view point of power factor as determined from these meter readings. In looking through the literature it was found that two Englishmen made a test on some single phase watt-hour meters about fifteen years ago. However, their data were of no value in the present research for two reasons; first, because present day meters may have been improved and second because one cannot be absolutely certain that three phase meters will act like single phase meters without trying them out experimentally. The question of phase sequence did not enter into the subject when dealing with single phase.

In order to make such a study it was necessary to have two or more alternators direct connected. Two Westinghouse alternators were geared together for this purpose, the gear ratio being three to one. These were 220 volt, 60 cycle, 1200 r.p.m. three phase machines with connections brought out making it possible to connect their windings in a number of ways. These two alternators were driven by a synchronous motor the pulley ratio being such that they run at 500 and 1500 r.p.m. Under these conditions the frequency obtained was 25 and 75 cycles per second. The machine running at 1500 r.p.m. had its windings connected in parallel wye, as shown in the diagrams on page forty-five. The machine running at 500 r.p.m. had its windings connected in series as shown in the diagrams. In this manner it was possible to operate either generator at any voltage from zero to 115 volts. This made it possible to obtain any desired phase relation between harmonic and fundamental. One of the machines had a handwheel for this purpose but unfortunately this machine had to be used for the third harmonic generator and the handwheel gave a shift of only about thirty degrees on the fundamental scale. This however, was no serious matter since it was a simple matter to loosen one machine, lift it with the crane and thereby change the position of the gear teeth.

The Sangamo Electric Company of Springfield Illinois very kindly agreed to send 25 cycle polyphase meters of their latest design for these tests. The tests were performed first on a watt-hour meter and then on a R.K.V.A.H. meter.

1. Kilowatt-hour Meters.

The watt-hour meter was first calibrated on a sine wave produced by a sine wave alternator. The accuracy was as shown on page forty four. It is of course impossible to use a rotating standard in a test of this kind because there is every reason to believe that the standard would be subject to the same inaccuracies upon non-harmonic waves. A new Weston polyphase wattmeter was therefore used for this purpose. This meter contains no iron in its magnetic circuit and is guaranteed accurate on frequencies up to one thousand cycles per second. It should be noted that the error at or near full load was a small fraction of one per cent on the sine wave.

The meter was then subjected to non-sinusoidal waves with the results shown in the curves on page forty-four.

2. Reactive-Kilovolt-Ampere-Hour Meters.

A R.K.V.A.H. meter for polyphase measurements consists of a K.W.H. meter and a phasing transformer, the latter for the purpose of displacing the voltage applied to the potential coils of the meter by ninety degrees from the line voltage. Connection diagrams and a vector diagram will be found on page forty-seven.

Referring now to the vector diagram, it will be seen that for unity power factor $E_{B''A''}$ and I_{aA} give a positive torque while $E_{B''C''}$ and I_{cC} give an equal negative torque and the meter reads zero. If the current is slightly lagging, the torque in a positive direction is increased and the torque in the negative direction decreased with the result that the meter reads positively on inductive load when the phase rotation is ABC.

Now let us consider what would happen if the phase sequence were reversed. The easiest way to do this is to use the same vector diagram but rotate vectors clockwise instead of counter clockwise. It will be seen at once that the position of all vectors is just the same as before but E_{CA} now lags 120 degrees behind E_{AB}. In other words the phase rotation is now ACB. For counter clockwise rotation of vectors

I_{aA} leads $E_{B'A'}$ and I_{cC} leads $E_{B''C''}$. For clockwise rotation of vectors I_{aA} lags $E_{B'A'}$ and I_{cC} lags $E_{B''C''}$. Now let the current be slightly lagging. The angle between $E_{B'A'}$ and I_{aA} will increase and this element will produce less and less positive torque as I_{aA} approaches the 90 degree position. During this time however I_{cC} approaches the 180 degree position and therefore this element provides more and more negative torque which will be a maximum when I_{cC} lags $E_{B''C''}$ by 180 degrees. The resultant torque is therefore seen to be negative and the meter runs backwards.

Now let us consider the importance of the foregoing facts and theory. Let the third harmonic have phase rotation opposite to that of the fundamental. The fundamental current and voltage then produce positive rotation of the disk while the 3rd harmonic produces negative rotation. The net result will depend upon the relative values of currents, voltages, and phase displacements. The largest error in the reactive component reading will occur for loads which are only slightly inductive. The reason for this is clear when it is remembered that for non-inductive load the meter reads zero. For very small angles of lag the net torque produced by the fundamental of current and voltage is almost zero but since the load reactance to the third harmonic is three times its value for the fundamental the third harmonic of current will lag its voltage by a much larger angle. Since this produces a negative torque, it is obvious that for small angles (less than five degrees) the meter may fail entirely to respond. This would be the case when the negative torque balances the positive torque. The load is inductive b t the power factor as determined by the R.K.V.A.H. meter and K.W.H. meter is unity.

The question arises--would we be justified in stating that the error in the reading of the meter is 100% or two hundred per cent? The answer to this question depends upon our definition of "reactive power". On page 259 of the April 1933 issue of Electrical Engineering will be found in large black print the question; "what is reactive power?" This is a subject which is receiving much attention at the present time. Although many pages are devoted to the subject, the fact remains that the question is not answered. An answer to that question does not come under the title of this thesis. However, regardless of the many views of the subject, any satisfactory definition must be one which will make it possible to put reactive power on a dollars and cents basis. A certain amount of equipment is tied up, serving no useful purpose. A asis of charges must be provided and apportioned which will provide interest on this investment. The amount of equipment in use depends upon the effective value of the current. When harmonics are present the effective value of the current is the square root of the sum of the squares of the effective values of the harmonics. <u>It does not</u> depend upon the relative phase sequence of the harmonics and fundamental.

3. Results and Discussion.

(a). Watt-hour Meters.

It was found that the phase rotation had no bearing on the reading whatever. Likewise the displacement between the fundamental and the harmonic had no effect on the reading of the meter. Furthermore, the accuracy of the meter was very good under full load. At light loads the accuracy was not as high as under load conditions, the error running as high as six per cent. See curve sheet on page 44 for the distribution of these errors with the various loads.

(b). R.K.V.A.H. Meters.

Although the phase sequence has no bearing upon the operation and accuracy of the watt-hour meter it does have a very decided effect upon the reading of a R.K.V.A.H. meter. Hence, when such a meter is used to determine power factor it will give results which are accurate only when used upon a sine wave. This has now been shown to be true for the transmission line considered in the appendix; it has been shown to be true experimentally in the laboratory by actually applying a voltage wave containing a third harmonic of phase sequence opposite to that of the fundamental; finally a theoretical explanation has been provided which makes it possible to anticipate the accuracy of the meter and which checks experimental data.

TABLE VIII

Calibration Data for Watt-Hour Meter on Sine Wave

Current	True Watts	Watts read	% Error	
9.3	1840	1835	-.272	
9.3	1828	1820	-.438	
9.3	1832	1807	-1.360	
9.3	1828	1807	-1.150	
9.3	1812	1807	-.280	Average Error -.70
8.0	1580	1585	-.316	
8.0	1580	1560	-1.267	
8.0	1572	1575	.190	
8.0	1580	1550	-1.900	
8.0	1570	1565	-.318	Average Error -.57
7.06	1428	1405	-1.610	
7.06	1412	1395	-1.210	
7.06	1404	1390	-.997	
7.06	1408	1395	-.923	
7.06	1388	1390	.144	Average Error -.92
4.04	924	930	.650	
4.04	920	923	.326	
4.04	912	923	1.206	
4.04	912	913	.110	
4.04	904	911	.774	Average Error .656

TABLE VIII Continued

1.83	390	393	.763
1.83	390	390	.000
1.83	386	393	1.815
1.83	386	390	1.065
1.83	386	384	-.518 Average Error .620
1.09	214	216	.934
1.09	210	214	1.905
1.09	210	214	1.905
1.09	212	215	1.415
1.09	211	214	1.420 Average Error 1.510

TABLE IX

Calibration Data for Watt-Hour Meter on Full Load. (Non-Sinusoidal Waves)

Current	True Watts	Watts Read	% Harmonic	% Error	
9.4	1828	1821	0	-.383	
9.4	1800	1790	0	-.555	
9.4	1788	1775	0	-.727	
9.4	1772	1763	0	-.507	Average Error
9.4	1764	1756	0	-.453	-.51
9.4	1840	1840	20	.000	
9.4	1828	1816	20	-.655	
9.4	1832	1817	20	-.817	
9.4	1828	1817	20	-.602	Average Error -.448
9.4	1820	1817	20	-.165	
9.4	1840	1833	40	-.381	
9.4	1844	1844	40	.000	
9.4	1848	1830	40	-.975	
9.4	1848	1844	40	-.216	Average Error
9.4	1840	1830	40	-.543	-.429
9.4	1912	1913	60	.052	
9.4	1912	1890	60	-1.150	
9.4	1904	1896	60	-.420	
9.4	1912	1890	60	-1.150	Average Error
9.4	1900	1890	60	-.530	-.656

TABLE IX Continued

9.4	1868	1832	80	-1.93	
9.4	1872	1840	80	-1.71	
9.4	1872	1833	80	-2.62	
9.4	1872	1840	80	-1.71	Average Error
9.4	1880	1833	80	-2.50	-2.09
9.4	1828	1764	100	-3.49	
9.4	1848	1764	100	-4.55	
9.4	1832	1764	100	-3.71	
9.4	1828	1764	100	-3.50	Average Error
9.4	1832	1770	100	-3.38	-3.72

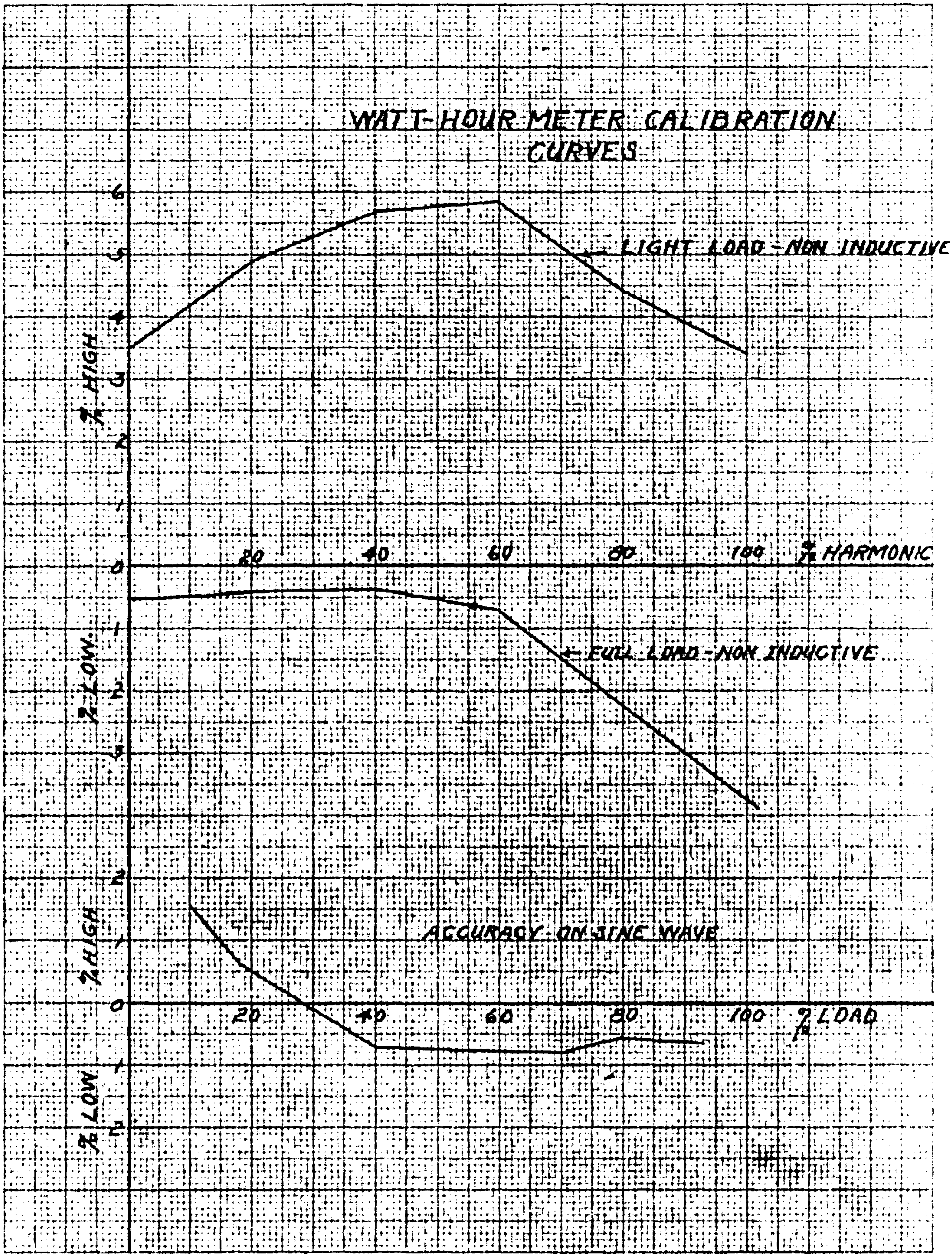
TABLE X

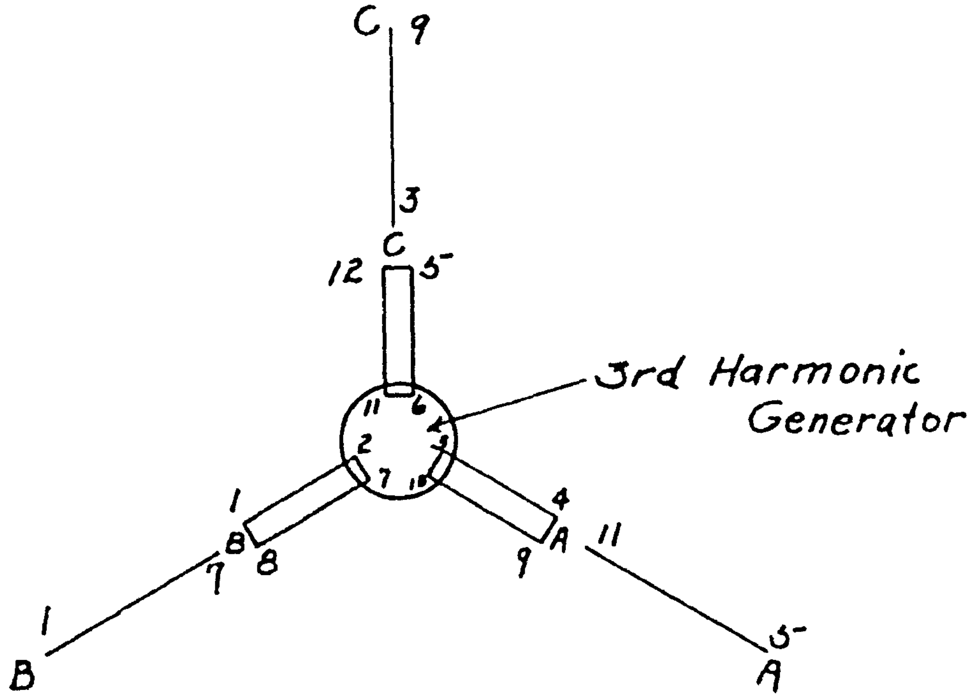
Calibration Data for Watt-Hour Meter on Light Load. (Non-sinusoidal Waves)

Current	True Watts	Watts Read	% Harmonic	%Error	
1.09	202	209	0	3.46	
1.09	201	207	0	2.98	
1.09	200	207	0	3.50	
1.09	200	207	0	3.50	Average Error
1.09	198	203	0	2.53	3.20
1.09	210	218	20	3.81	
1.09	210	221	20	5.23	
1.09	208	218	20	4.80	
1.09	208	217	20	4.32	Average Error
1.09	208	219	20	5.30	4.69
1.08	211	223.5	40	5.93	
1.08	211	223.0	40	5.68	
1.08	212	223.5	40	5.43	
1.08	212	222.5	40	4.95	Average Error
1.08	210	223.5	40	6.43	5.68
1.07	215	227	60	5.48	
1.07	214	228	60	6.52	
1.07	216	228.5	60	5.80	
1.07	214	228	60	6.53	Average Error
1.07	214	227	60	6.08	6.08

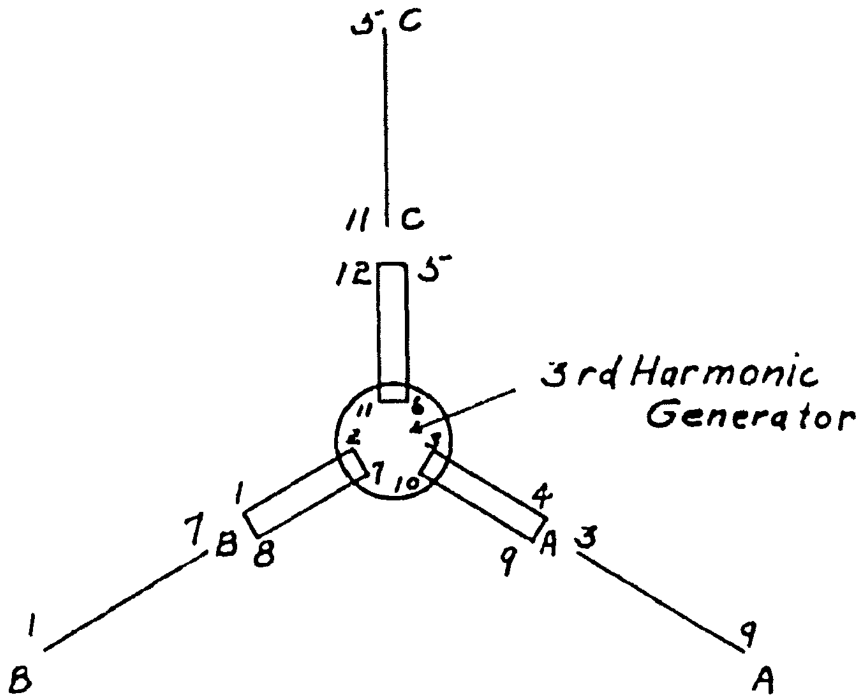
TABLE X Continued

1.06	217	226	80	4.15	
1.06	216	227	80	5.09	
1.06	217	228	80	5.06	
1.06	218	226	80	3.66	Average Error
1.06	217	226.5	80	4.37	4.47
1.05	216	223	100	3.24	
1.05	214	224	100	4.47	
1.05	216	221.5	100	2.55	
1.05	216	225	100	4.17	Average Error
1.05	216	222.5	100	3.00	3.49

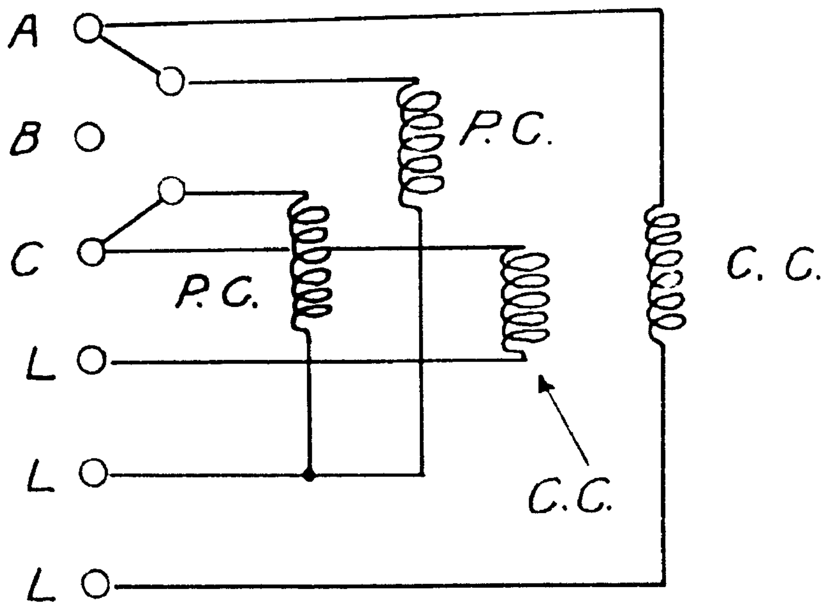

WATT-HOUR METER CALIBRATION CURVES
LIGHT LOAD - NON INDUCTIVE
FULL LOAD - NON INDUCTIVE
% HARMONIC
% HIGH
% LOW
ACCURACY ON SINE WAVE
% LOAD
% HIGH
% LOW
20
40
60
80
100
0
1
2
3
4
5
6



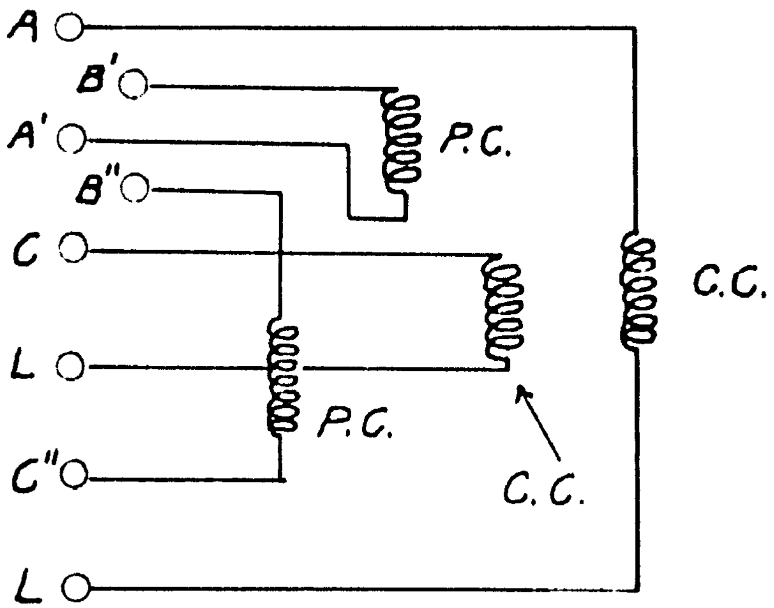
GENERATOR CONNECTIONS
SAME PHASE ROTATION



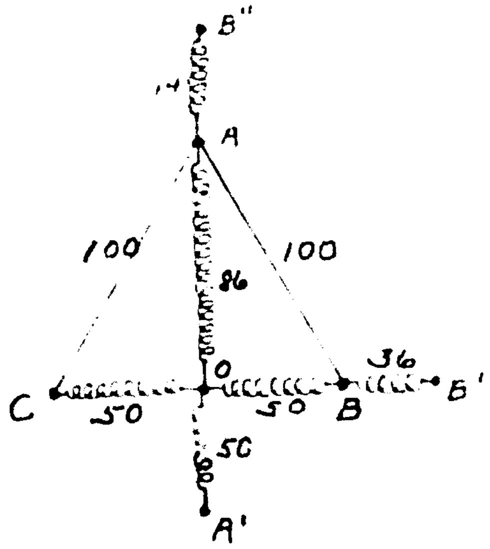
GENERATOR CONNECTIONS
OPPOSITE PHASE ROTATION



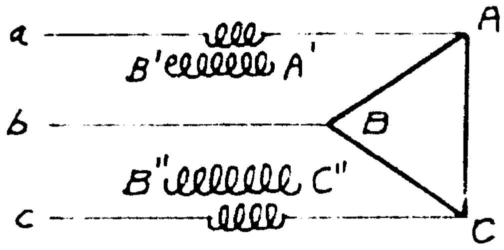
CONNECTION DIAGRAM
K.W.H. METER AS RECEIVED



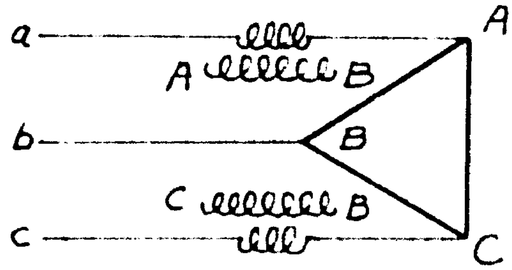
CONNECTION DIAGRAM
R.K.V.A.H. METER AFTER MAKING CHANGES



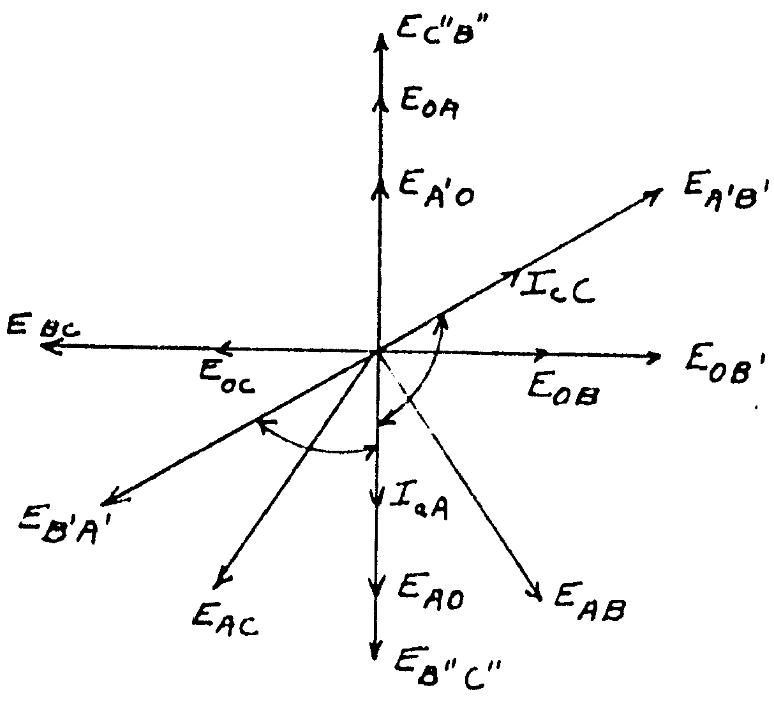
PHASE SHIFTING
TRANSFORMER



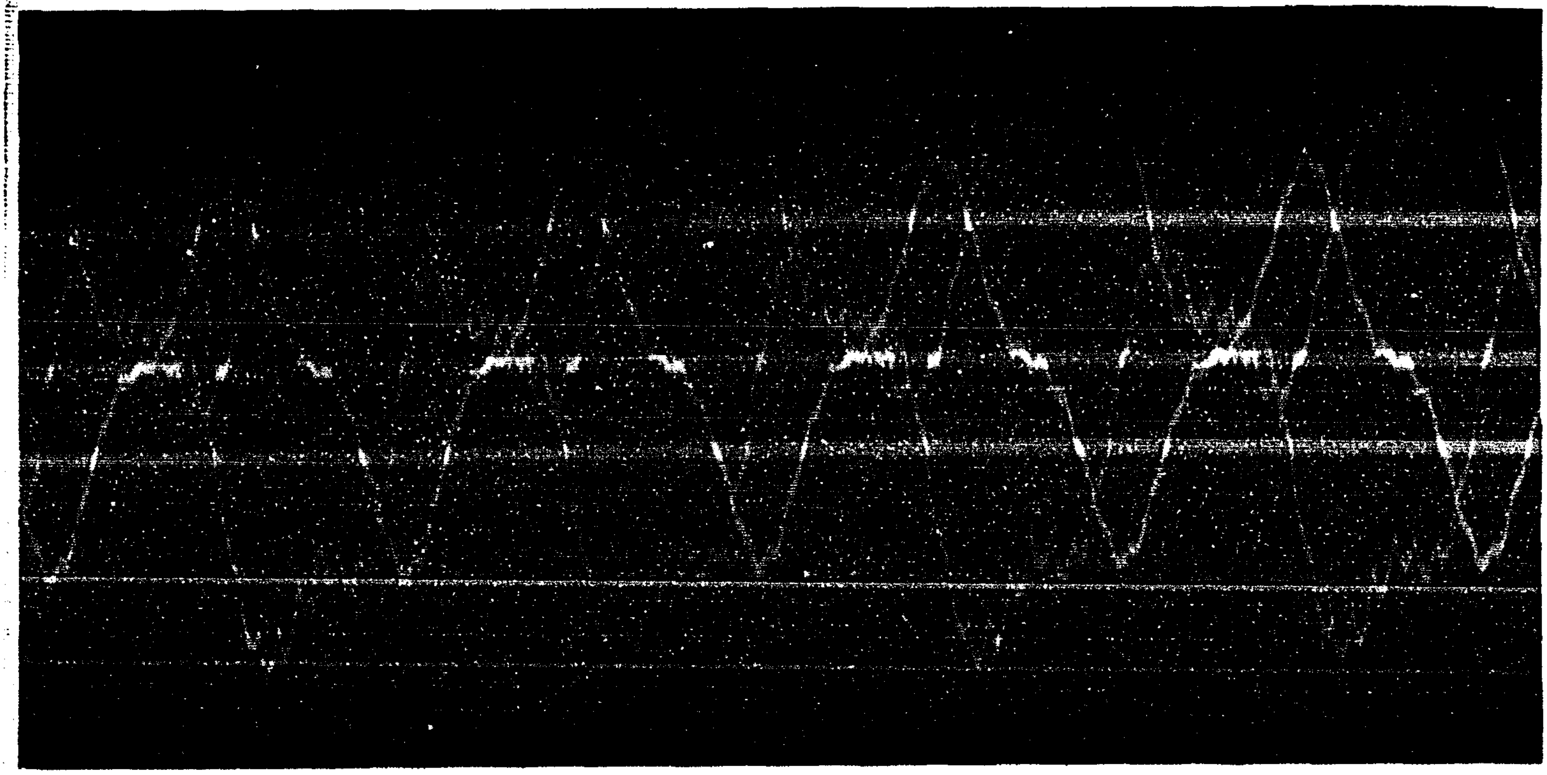
REACTIVE METER



WATT-HOUR METER

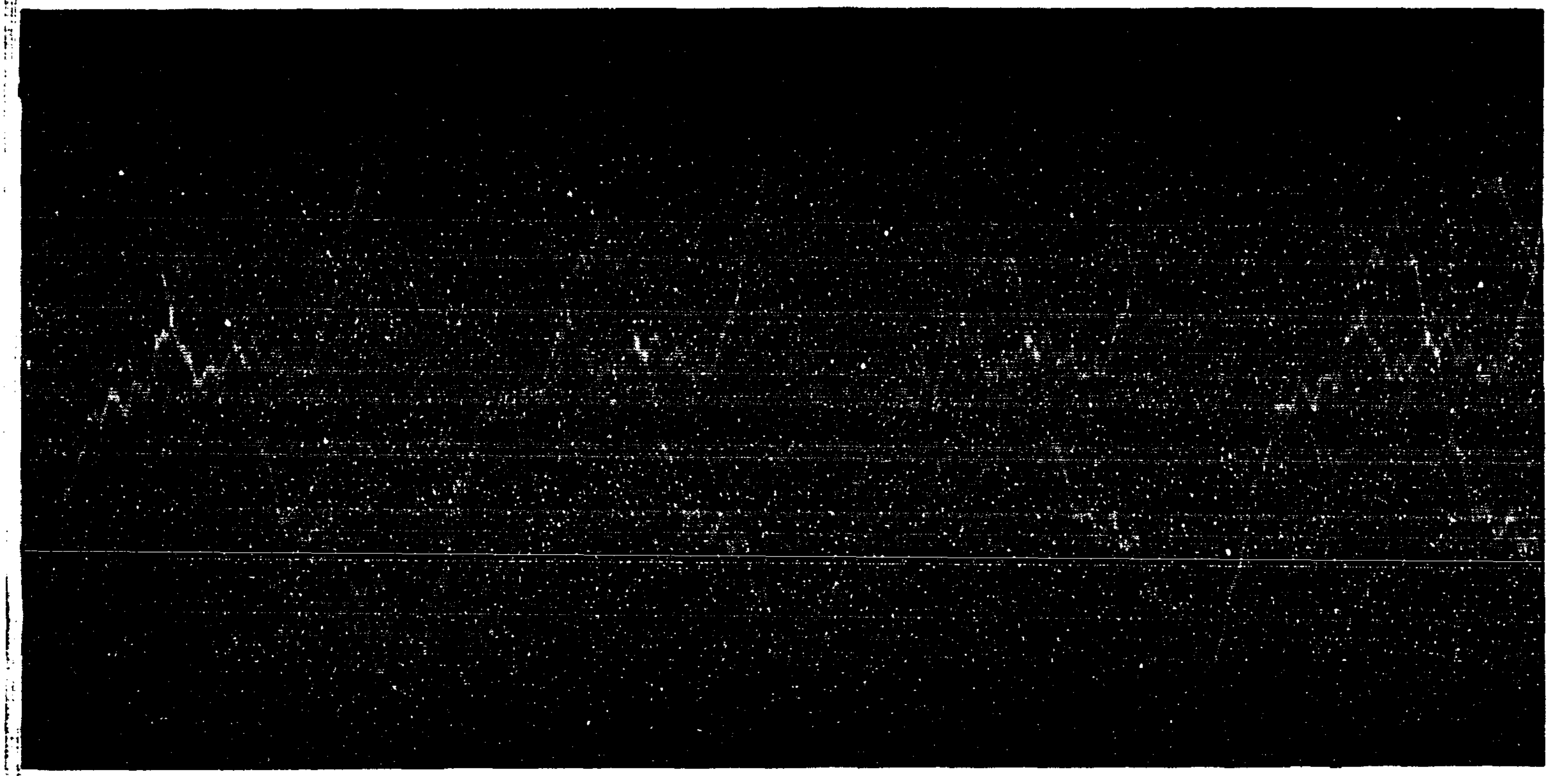


VECTOR DIAGRAM FOR R.KV.A.H. METER



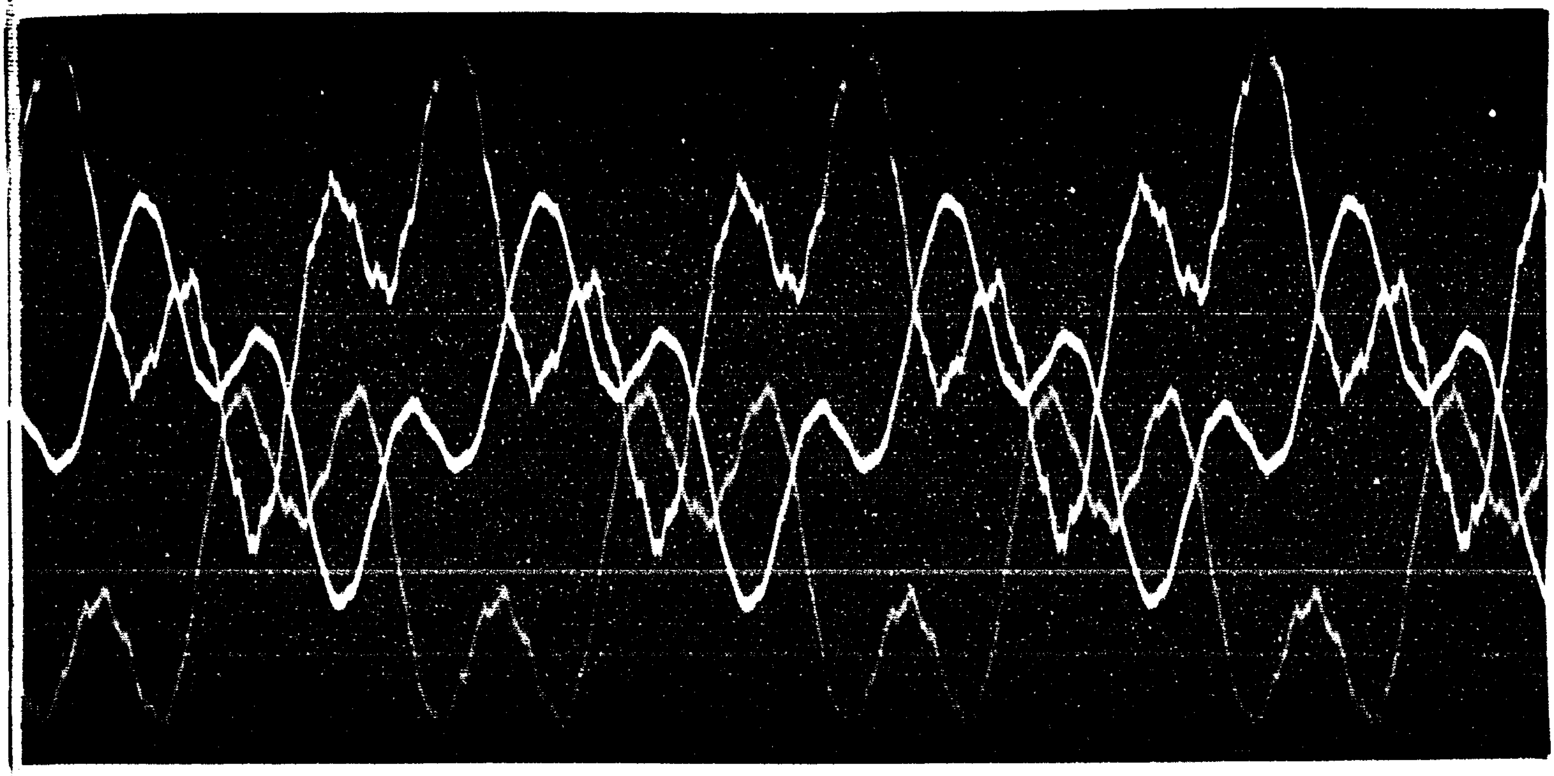
Line Voltages

30% 3rd Harmonic--Same Phase Rotation



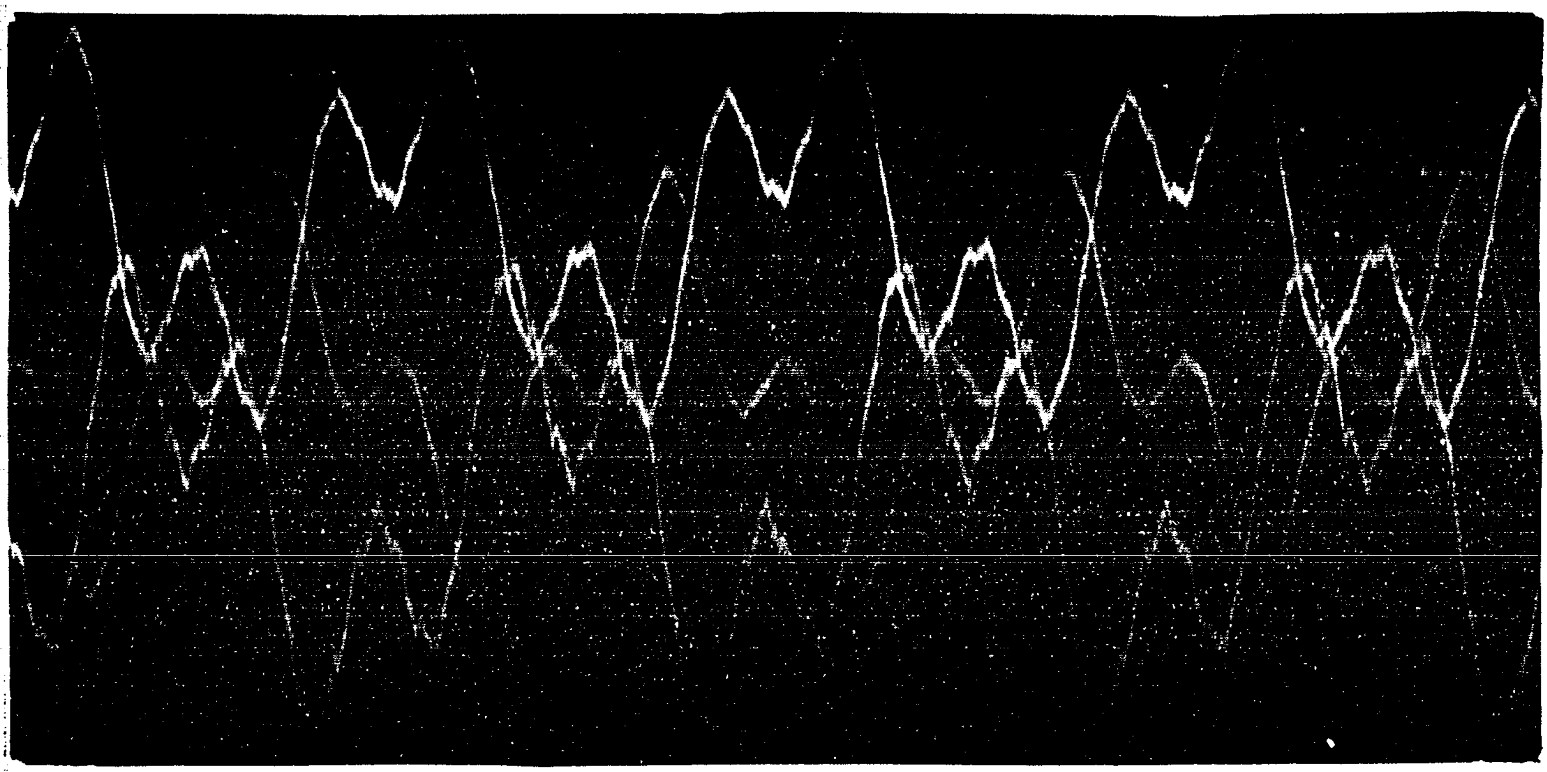
Line Voltages

30% 3rd Harmonic--Opposite Phase Rotation



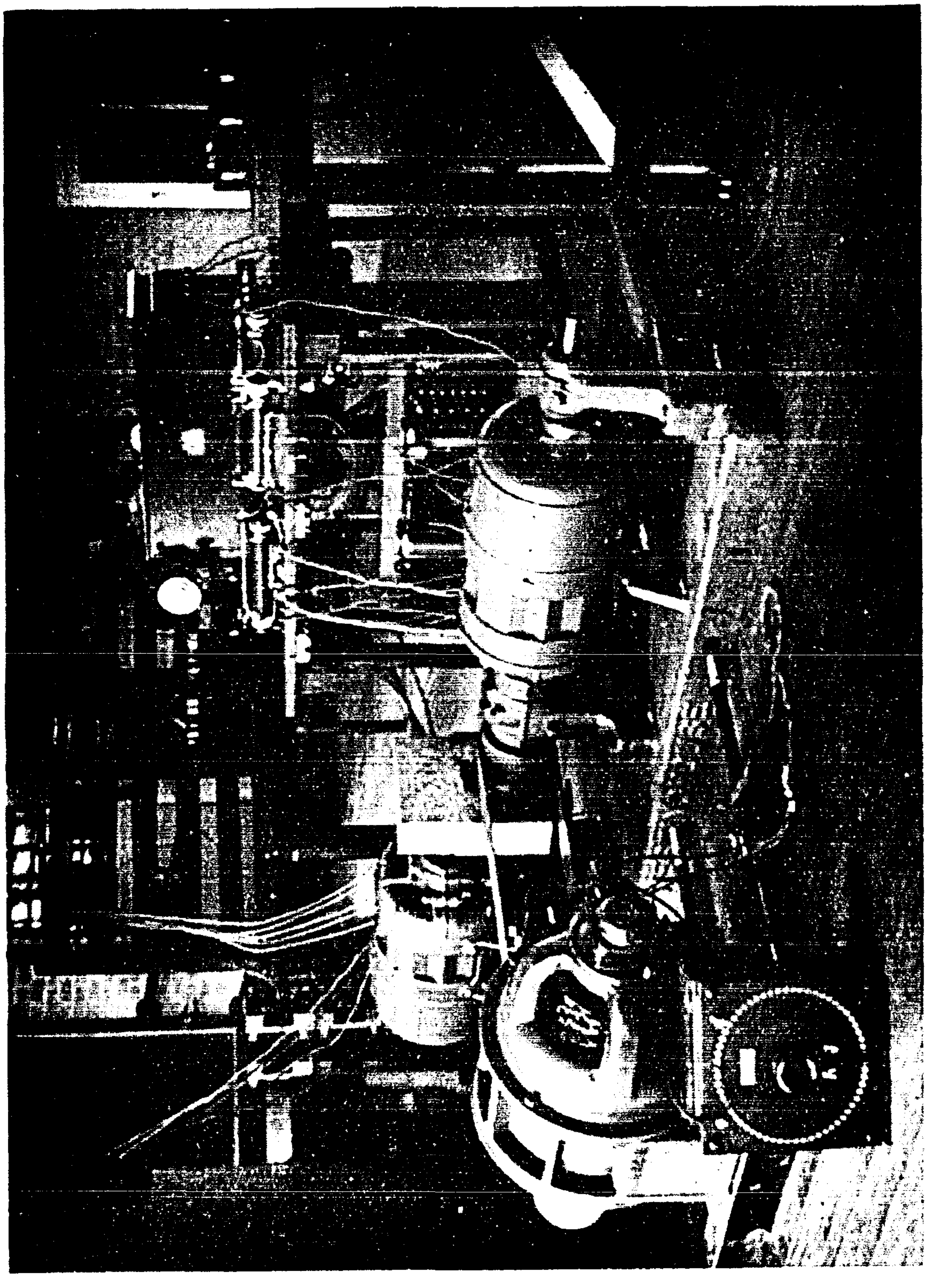
Line Voltages

80% 3rd Harmonic--Same Phase Rotation



Line Voltages

80% 3rd Harmonic--Opposite Phase Rotation



VI THE OPERATION OF INDUCTION MOTORS ON NON-SINUSOIDAL WAVES

1. 2nd Harmonic--Same Phase Rotation.

Tests were made on a Westinghouse Type C.W. 3 phase, 110 volt, 60 cycle, 4 pole, wound rotor, induction motor. A second harmonic was obtained in the voltage wave being applied to the motor by means of a Westinghouse type C.W. frequency changer and three 2 K.V.A. transformers connected as shown in the diagram page fifty five. The frequency changer was supplied at 60 cycles and gave out a voltage whose frequency was 120 cycles per second. The transformer ratio was 4 to 1 for each transformer. The voltage across the secondary of each transformer due to the 120 cycle source was 38 volts while the line voltage due to the 60 cycle source was 115 volts. This gave a line voltage due to the resultant wave of 132 volts, the harmonic being fifty seven per cent of the fundamental. Under these conditions using the same phase rotation the motor had a no load speed of 1800 r.p.m. In other words the motor operated at synchronous speed. However, it should be noted that under these conditions the frequency of the rotor current was 60 cycles per second instead of the usual three or four. (See oscillogram # 3, page 60) The effective value of the rotor current at no load was 7.8 amperes as compared with .32 amperes when a sine wave of voltage was applied. The power required to operate the motor at no load was about two and a half times the amount required when a sine wave was applied.

2. 2nd Harmonic--Opposite Phase Rotation.

In the case of opposite phase rotation the rotor current had a frequency of about 180 cycles per second. Actually, of course the frequency was slightly less by the amount of the slip. The motor ran slower than its normal no load speed. There was also considerable vibration and a slight tendency to growl. The power used was 645 watts, about four times the amount needed when a sine wave was applied. The oscillogram (No.2 page 59) showed no low frequency component in the rotor current for the reason that the low frequency component was of the order of one half ampere as compared with a current of 9.55 amperes.

3. 3rd Harmonic--Same Phase Rotation.

A third harmonic was obtained in the voltage wave supplied to the motor under test in the manner shown in the diagram on page fifty six. A wound rotor induction motor was driven at its rated speed and supplied with power from a 120 cycle 220 volt source. By driving it in the direction opposite to that which it would have if allowed to run as an induction motor, a source of 180 cycles was obtained. In this case the magnitude of the third harmonic voltage inserted into the line was 23 volts. The fundamental was 115 volts as before. This gave a line voltage of 117 volts and a harmonic which was 34.5 per cent of the fundamental.

The power necessary to operate the motor at no load was about one and one half times the amount required for a sine wave.

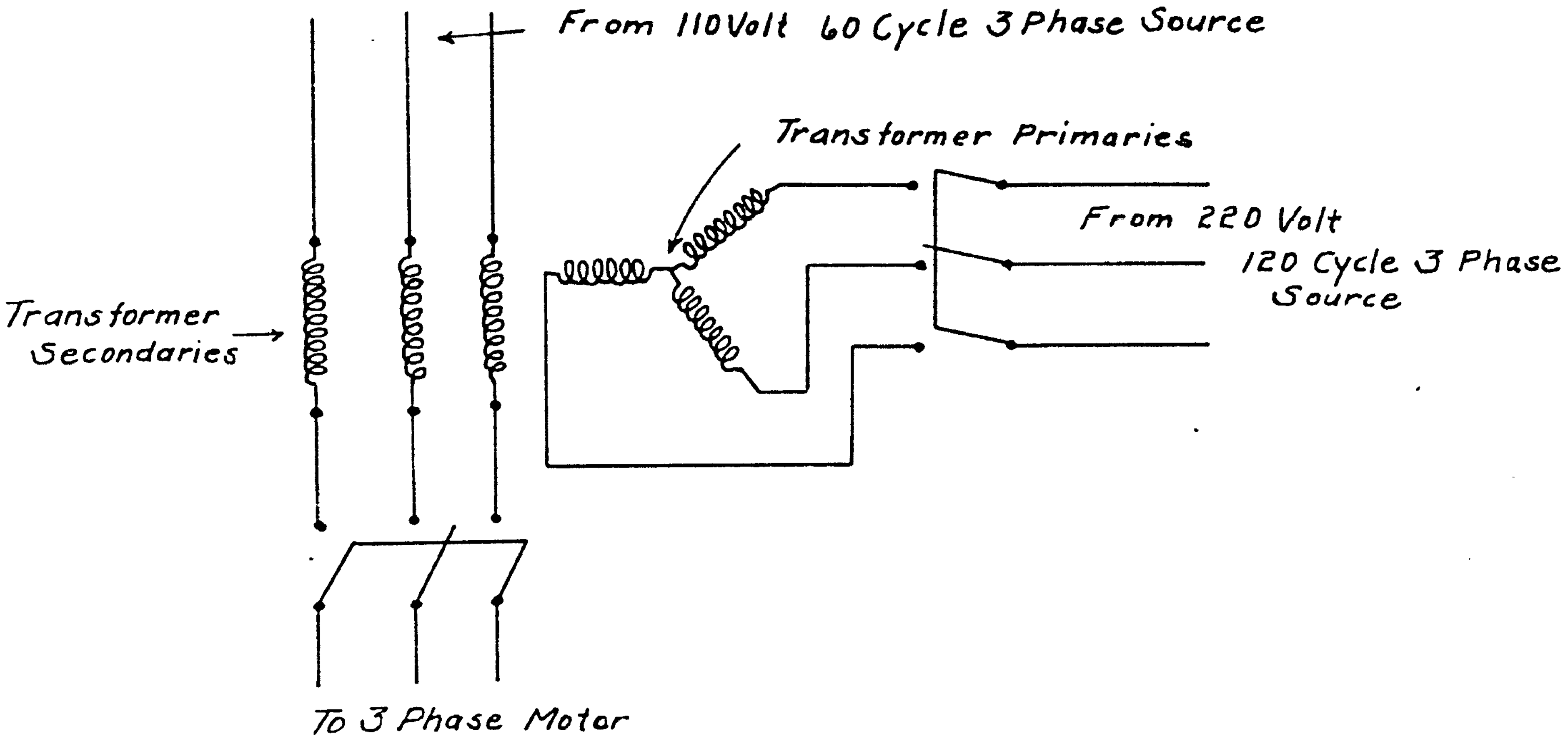
4. 3rd Harmonic--Opposite Phase Rotation.

The operation of the motor for this case was not much different from theone preceding. However, the motor vibrated more and had a rasping sound, making sixty two of these per minute as timed by a stop watch. This noise stopped immediately upon removing the third harmonic.

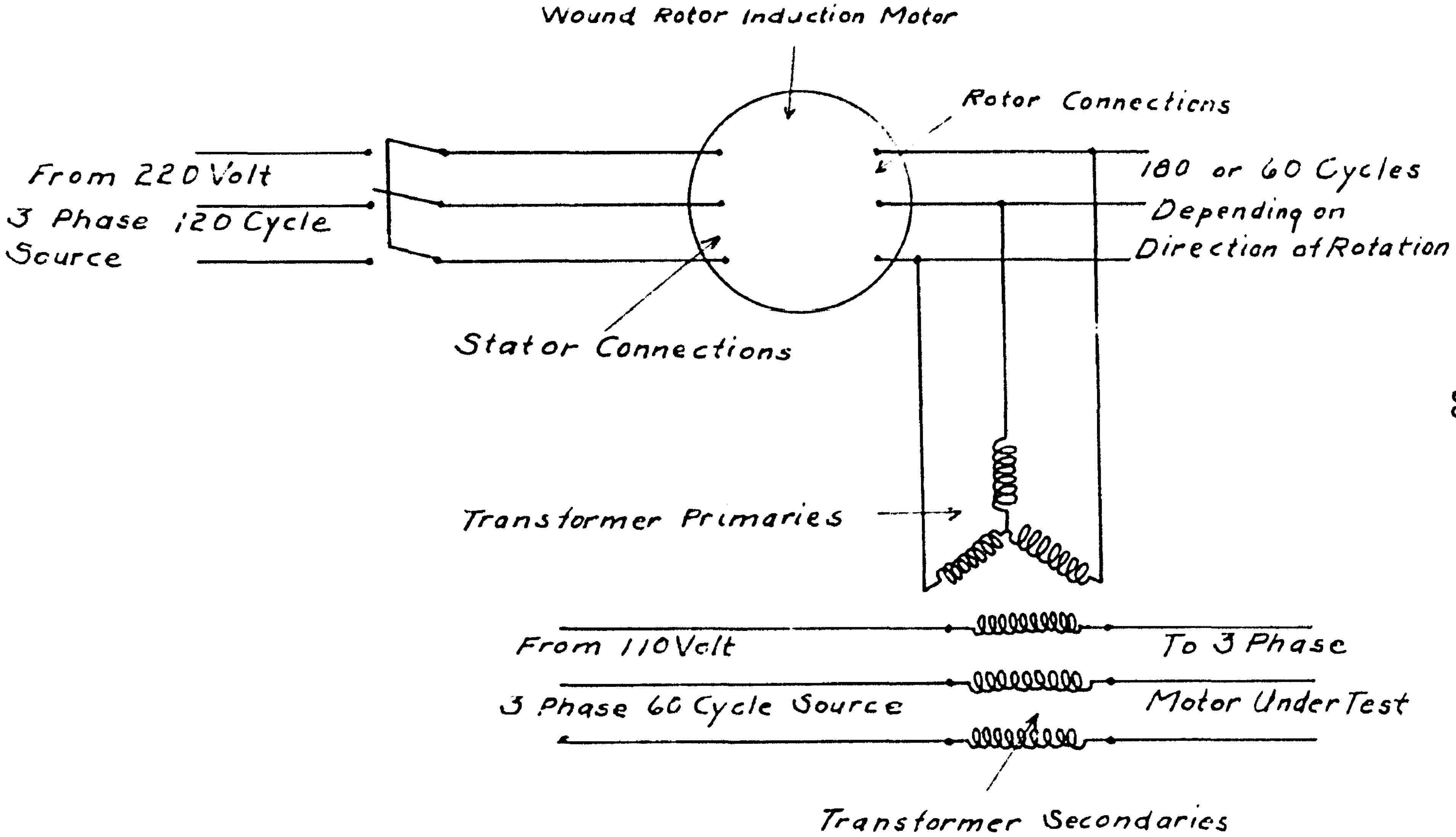
5. Results and Discussion.

General results followed more logically the discussion for the separate harmonics and have therefore been taken up in that manner.

Looking at film #4,page 61, it will be noticed that the rotor current frequency was 112½ cycles. For the same phase rotation this should be 120 cycles if the frequency of the harmonic is exactly 180 cycles per second. The fact that it was 112½ cycles may be accounted for in a number of ways. If the frequency from the power house dropped say to 58, the frequency converter would deliver a voltage whose frequency was 116 cycles. This would then be supplied to the stator of the wound rotor induction motor which probably run at a slip corresponding to 2 cycles or fifty six. Driving the motor in the opposite direction to that it would have if operated as an induction motor would give a frequency of 172 cycles per second. When this is supplied to the ½ H.P. motor using the same phase sequence for the fundamental and harmonic the motor has extra driving torque bringing its speed up to about 1800 r.p.m. This corresponds to that it would have if the frequency were 60 cycles per second. Hence the frequency in the rotor would be 172 minus 60 or 112 cycles per second.



SHOWING METHOD OF OBTAINING 2ND. HARMONIC

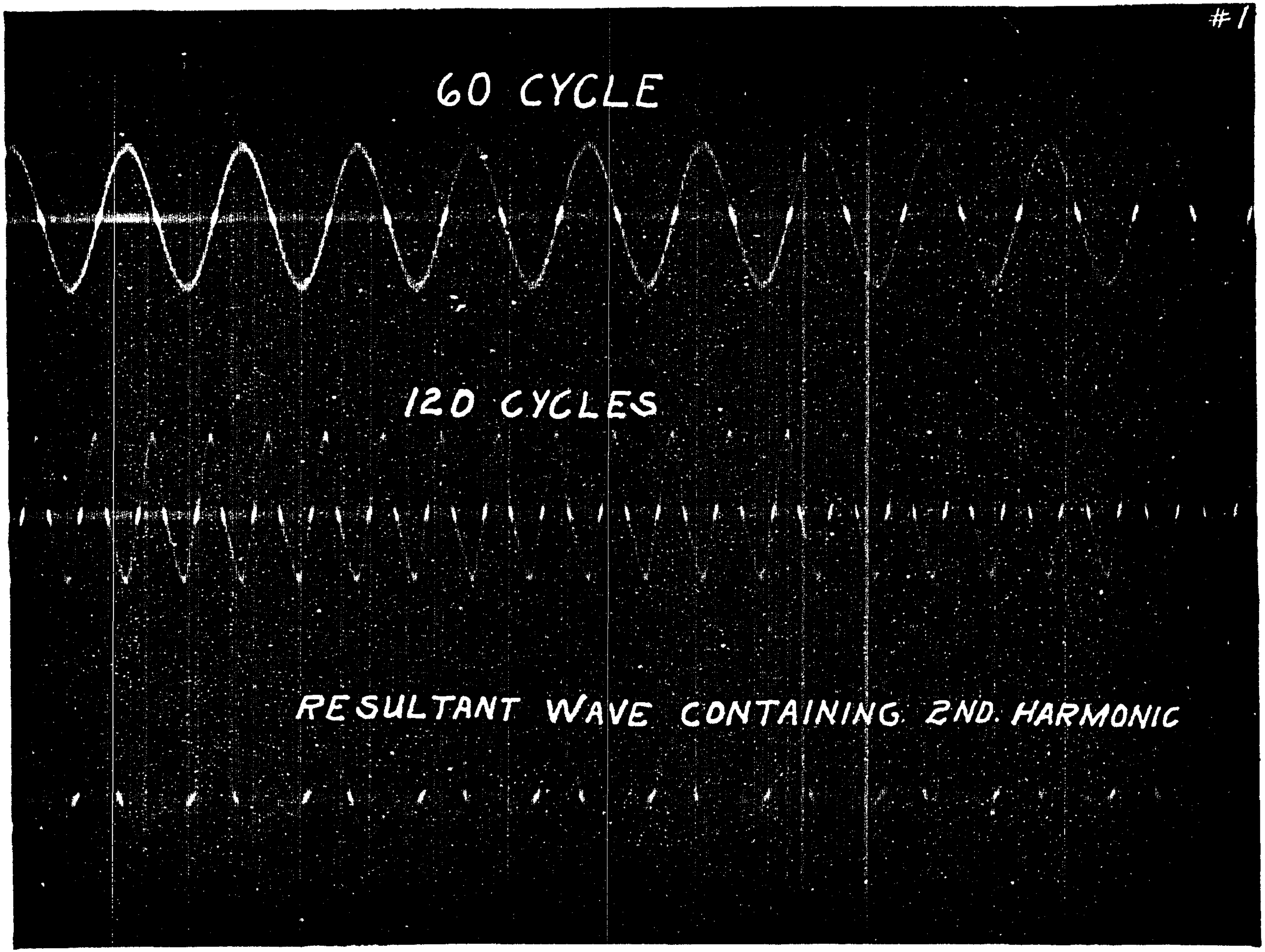


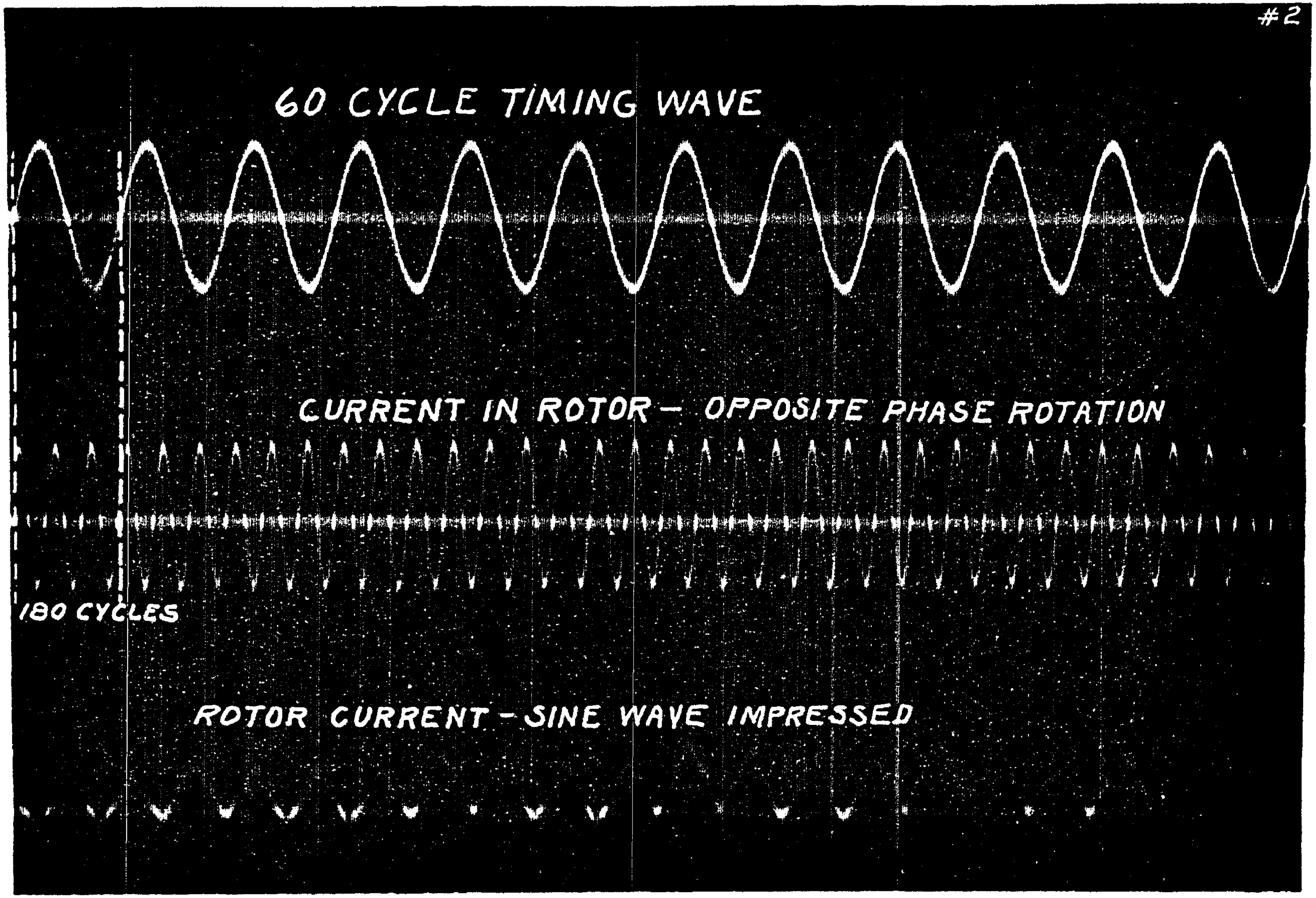
SHOWING METHOD OF OBTAINING 3RD. HARMONIC

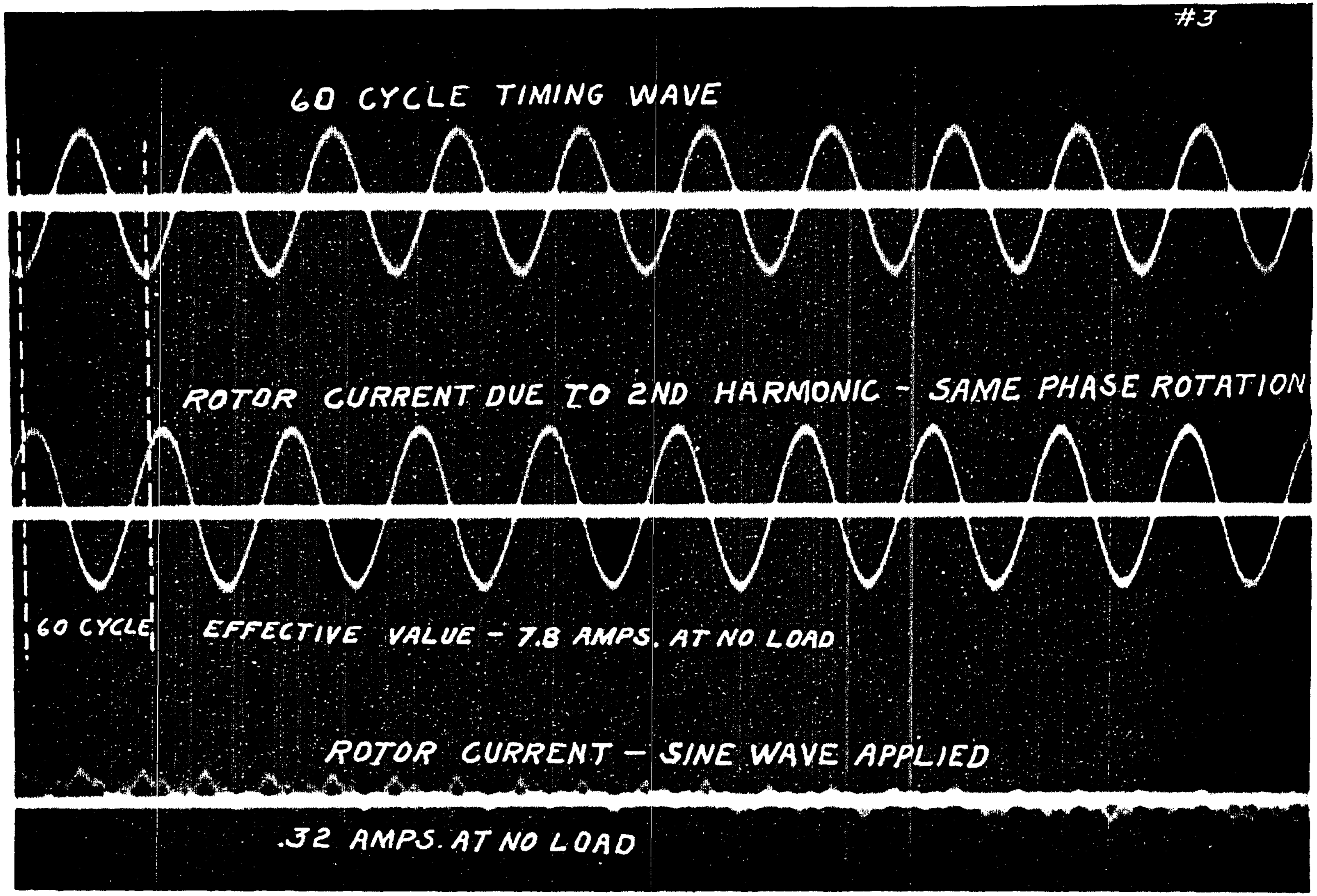
TABLE XI

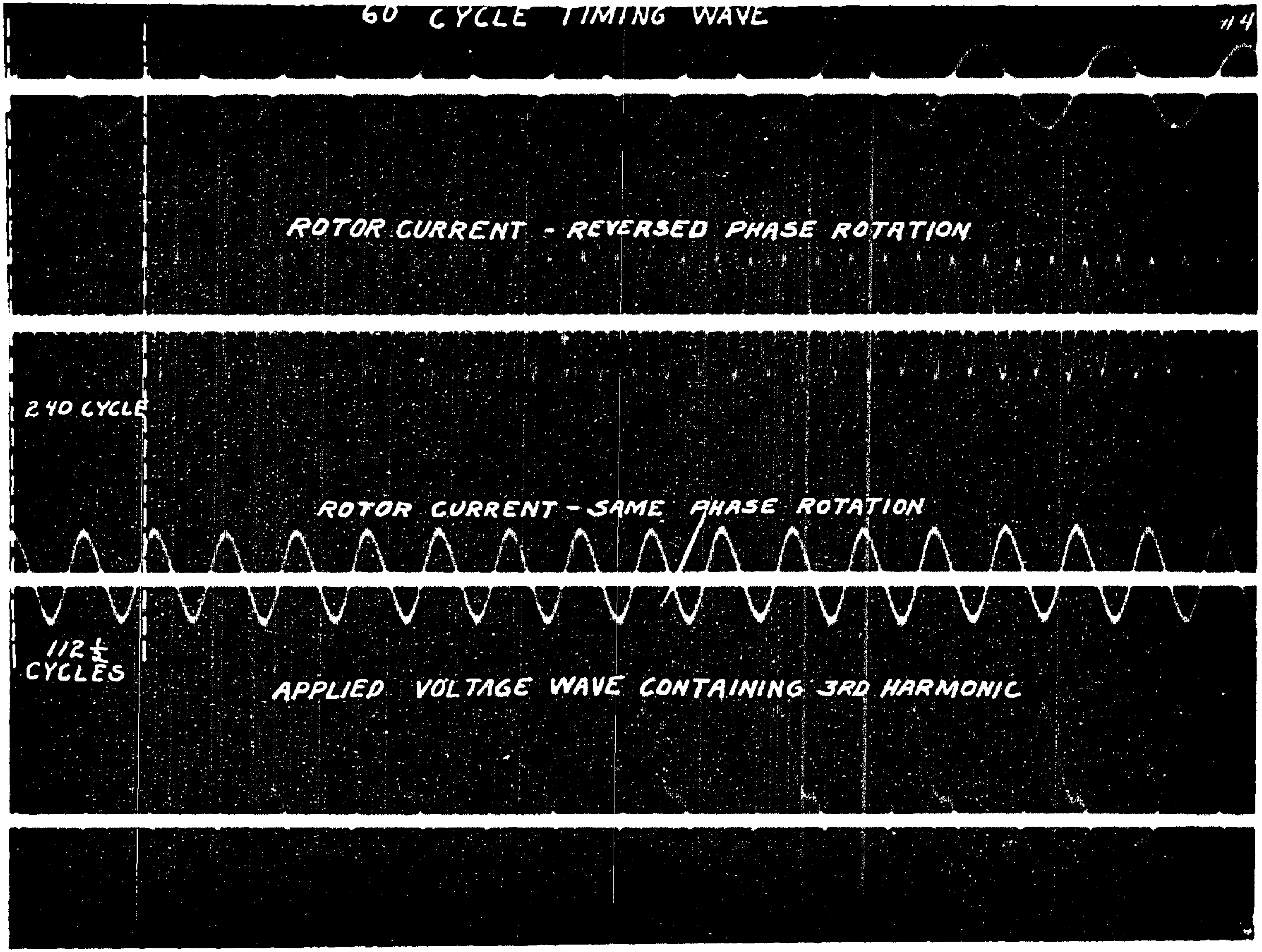
Induction Motor Operating on Voltages Containing 2nd and 3rd Harmonics.

Film Number	Watts Input	Stator Current	Rotor Current	Line Volts	Phase Sequence	Order of Harmonic	Per Cent Harmonic
1-2	645	8.7	9.55	132	Opposite	2nd	57
3	490	7.8	7.5-8.5	132	Same	2nd	57
4	250	4.6	3.6-4.1	117	Same	3rd	34.5
4	260	4.65	3.9-4.3	117	Opposite	3rd	34.5


#1
60 CYCLE
120 CYCLES
RESULTANT WAVE CONTAINING 2ND. HARMONIC


#2
60 CYCLE TIMING WAVE
CURRENT IN ROTOR — OPPOSITE PHASE ROTATION
180 CYCLES
ROTOR CURRENT - SINE WAVE IMPRESSED


#3
60 CYCLE TIMING WAVE
ROTOR CURRENT DUE TO 2ND HARMONIC - SAME PHASE ROTATION
60 CYCLE
EFFECTIVE VALUE - 7.8 AMPS. AT NO LOAD
ROTOR CURRENT - SINE WAVE APPLIED
.32 AMPS. AT NO LOAD


60 CYCLE TIMING WAVE
#4
ROTOR CURRENT - REVERSED PHASE ROTATION
240 CYCLE
ROTOR CURRENT - SAME PHASE ROTATION
112½ CYCLES
APPLIED VOLTAGE WAVE CONTAINING 3RD HARMONIC

VII APPENDIX

1. General Equations.

The following equations give the values of current and voltage at any point on a transmission line when a sine wave of voltage is applied at the sending end.

$$E = \frac{E_o}{2}\left[\epsilon^{\sqrt{ZY}l} + \epsilon^{-\sqrt{ZY}l}\right] - \frac{I_o}{2}\sqrt{\frac{Z}{Y}}\left[\epsilon^{\sqrt{ZY}l} - \epsilon^{-\sqrt{ZY}l}\right]$$

and $$I = \frac{I_o}{2}\left[\epsilon^{\sqrt{ZY}l} + \epsilon^{-\sqrt{ZY}l}\right] + \frac{E_o}{2}\sqrt{\frac{Y}{Z}}\left[\epsilon^{\sqrt{ZY}l} - \epsilon^{-\sqrt{ZY}l}\right]$$

where

E = Voltage at any distance l from the load.

E = Load voltage.

$Z = r + jx$

$Y = g + jb$

r = Resistance in ohms per mile.

x = Reactance in ohms per mile.

g = Conductance in mhos per mile.

b = Susceptance in mhos per mile.

Letting $V = \sqrt{ZY}$ and $U = \sqrt{\frac{Z}{Y}}$ and rewriting the equations in terms of hyperbolic functions

$$E = E_o \cosh Vl + I_o U \sinh Vl$$

$$I = I_o \cosh Vl + \frac{E_o}{U} \sinh Vl$$

Further simplification gives

$$E = AE_o + BI_o$$

$$I = AI_o + CE_o$$

where

$E =$ Voltage at generator.

$I =$ Current at generator.

$E =$ Voltage at the load.

$I =$ Current at the load.

$B = U \sinh Vl = b_1 + jb_2$

$C = 1/U \sinh Vl = c_1 + jc_2$

2. Calculation of Auxiliary Constants.

These are the constants to be used in the formulae on the preceding page. The results of these calculations will be grouped together in Table A. The transmission line data will be given first and following this the calculations will be made for the auxiliary constants for all harmonics including the fundamental.

A. Transmission Line Data.

Length- 250 miles

Frequency- 60 cycles per second

Voltage- 220,000 volts

K.W.A.- 75,000

Size of wire- 636,000 c.m. aluminum cable steel reinforced

Spacing- equivalent 21' delta

$r = .147$ ohms per mile.

$x = .763$ ohms per mile at 60 cycles per second.

$b = 5.4$ micro-mhos per mile at 60 cycles per second.

$g = 0$

B. Calculations.

(a). Fundamental.

$$\log x = 9.8825245-10$$

$$\tan \theta_z = \frac{x}{r} \qquad \log r = 9.1673173-10$$

$$\log \tan \theta_z = 0.7152072$$

$$z = \frac{x}{\sin \theta_z} \qquad \log \sin \theta_z = 9.9920851-10$$

$$\log z = 9.8904394-10$$

$$\theta_z = 79^\circ\ 5'\ 40''$$

$$\log y = 4.7323938-10$$

$$\theta_z = 90^\circ$$

$$v^2 = zy \qquad 2 \log v = 4.6228332-10$$

$$2\,\theta_v = \theta_z + \theta_y \qquad \log v = 7.3114166-10$$

$$2\,\theta_v = 169^\circ\ 5'\ 40''$$

$$\theta_v = 84^\circ\ 32'\ 50''$$

$$\alpha = v \cos \theta_v \qquad \log \cos \theta_v = 8.9778394-10$$

$$\beta = v \sin \theta_v \qquad \log \sin \theta_v = 9.9980303-10$$

$$\log \alpha = 6.2892560-10$$

$$\log \beta = 7.3094469-10$$

$$\log \frac{360}{2\pi} = 1.7581230$$

$$\log \beta' = 9.0675699-10$$

$$\alpha = .00019465 \text{ radians}$$

$$\beta^\circ = .11683 \text{ degrees}$$

$$u^2 = \frac{z}{y} \qquad 2 \log u = 5.1580456$$

$$\log u = 2.5790228$$

$$2\,\theta_u = \theta_s - \theta_y \qquad 2\,\theta_u = -10^\circ\ 54'\ 20''$$

$$\theta_u = -\ 5^\circ\ 27'\ 10'' \qquad U = 379.3\,\underline{|-5^\circ 27' 10''}$$

$$\frac{1}{u} = \sqrt{\frac{Y}{Z}} \qquad 2 \log \frac{1}{u} = 4.8419544-10$$

$$\log \frac{1}{u} = 7.4209777-10 \qquad \frac{1}{U} = .0026\,\underline{|5^\circ 27'\ 10}$$

$$\log \alpha = 6.2892560-10$$

$$\log 1 = 2.3979400$$

$$\log \alpha 1 = 8.6871960-10$$

$$\log \beta_0 1 = 1.4655099$$

$$\alpha l = 0.048663$$

$$\beta^\circ l = 29^\circ\ 12'\ \ 33''$$

$$\log \cosh \alpha l = 0.000514$$

$$\log \sinh \alpha l = 8.678258-10$$

$$\log \cos \beta l = 9.9409402-10$$

$$\log \sin \beta l = 9.6884079-10$$

$$\log \cosh \alpha l \cos \beta l = 9.9414542-10 = \log a_1$$

$$\log \sinh \alpha l \sin \beta l = 8.3666659-10 = \log a_2$$

$$\log \cosh \alpha l \sin \beta l = 9.6890219-10$$

$$\log \sinh \alpha l \cos \beta l = 8.6191982-10 \qquad a_1 = .87389$$

$$a_2 = .023263$$

$$S = \sinh Vl \qquad US = B \qquad B = b_1 + jb_2$$

$$S = \sinh Vl = \sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l$$

$$\log \tan \theta_s = \log \cosh \alpha l \sin \beta l - \log \sinh \alpha l \cos \beta l$$

$$= 1.0698237$$

$$\log \cos \theta_s = 8.9285866-10 \qquad \theta_s = 85^\circ\ 8'\ \ 0''$$

$\log S = \log \sinh Vl = \log \sinh + \lvert \cos\beta \rvert \log \cos \theta_s$

$= 9.6906116-10$

$\theta_b = \theta_u + \theta_s = 79^\circ\ 40'\ 50''$

$\log U \sinh Vl = \log B = 2.2696244$

$B = 186.05 \quad \lvert 79^\circ\ 40'\ 50''$

$b_1 = 33.329$

$b_2 = 183.04$

$$\begin{aligned} \log \cos \theta_b &= 9.2531831-10 \\ \underline{\log B} &= \underline{2.2696344} \\ \log b_1 &= 1.5228175 \end{aligned}$$

$$\begin{aligned} \log \sin \theta_b &= 9.9929175-10 \\ \underline{\log B} &= \underline{2.2696344} \\ \log b_2 &= 2.2625519 \end{aligned}$$

$C = \frac{1}{u} \sinh Vl \quad \lvert \theta_s - \theta_n$

$\log c = \log s - \log u$

$= 7.1115988-10$

$C = .001293 \quad \lvert 90^\circ\ 35'\ 10''$

$$\begin{aligned} \log \cos \theta_c &= 8.0098497-10 \\ \underline{\log C} &= \underline{7.1115988-10} \\ \log c_1 &= 5.1214485-10 \\ c_1 &= -.000013237 \end{aligned}$$

$$\begin{aligned} \log \sin \theta_c &= 9.9999773-10 \\ \underline{\log C} &= \underline{7.1115988-10} \\ \log c_2 &= 7.1115761-10 \\ c_2 &= .0012929 \end{aligned}$$

(b). <u>3rd Harmonic.</u>

$r = .147$ ohms per mile.

$x = 2.289$ ohms per mile.

$b = 16.2 \times 10^{-6}$ mhos per mile.

$g = 0$

$\log x = .4608978$

$\log r = 9.1673173-10$

$\log \tan \theta_z = 1.2935805$

$\log \sin \theta_z = 9.9994392-10$

$z = 2.893 \underline{/87^\circ\ 5'\ 20''}$ $\qquad \log z = 0.4614586$

$\theta_z = 87^\circ\ 5'\ 20''$

$\log y = 5.2095150-10$

$\theta_y = 90^\circ$

$v^2 = zy$ $\qquad 2 \log v = 5.6709736-10$

$2\,\theta_v = \theta_z + \theta_y$ $\qquad \log v = 7.8354868-10$

$2\,\theta_v = 177^\circ\ 5'\ 20''$

$\alpha = v \cos \theta_v$ $\qquad \theta_v = 88^\circ\ 32'\ 40''$

$\beta = v \sin \theta_v$ $\qquad \log \cos \theta_v = 8.4048594-10$

$\log \sin \theta_v = 9.9998598-10$

$\log \alpha = 6.2403462-10$

$\log \beta = 7.8353466-10$

$\log \frac{360}{2\pi} = 1.7581230$

$\log \beta^\circ = 9.5934696-10$

$$\alpha = .00017391$$

$$\beta^\circ = .39211$$

$$u^2 = \frac{z}{y} \qquad 2 \log u = 5.2519436$$

$$\log u = 2.6259718$$

$$2\theta_u = \theta_z - \theta_y \qquad 2\theta_u = -2^\circ\ 54'\ 40''$$

$$\frac{1}{u} = \sqrt{\frac{y}{z}} \qquad \theta_u = -1^\circ\ 27'\ 20''$$

$$\frac{1}{U} = .002366\underline{/1^\circ 27' 20''}$$

$$2 \log \frac{1}{u} = 4.7480564-10$$

$$\log \frac{1}{u} = 7.3740282-10$$

$$U = 422.75\underline{/-1^\circ 27'\ 20''}$$

$$\log \alpha = 6.2403462-10$$

$$\log l = 2.3979400$$

$$\log \alpha l = 8.6382862-10$$

$$\log \beta^\circ l = 1.9914096$$

$$\alpha l = 0.04348$$

$$\beta^\circ l = 98.041 \text{ degrees}$$

$$\log \cosh \alpha l = 0.000409$$

$$\log \sinh \alpha l = 8.6386020-10$$

$$\log \cos \beta l = -9.1435553-10$$

$$\log \sin \beta l = 9.9957528-10$$

$$\log \cosh \alpha l \cos \beta l = -9.1439643-10 = \log a_1$$

$$\log \sinh \alpha l \sin \beta l = 8.6343548-10 = \log a_2$$

$$\log \cosh \alpha l \sin \beta l = 9.9961618-10 \qquad a_1 = -.1393$$

$$\log \sinh \alpha l \cos \beta l = -7.7821573-10 \qquad a_2 = .043088$$

$$B = US = b_1 + jb_2 \qquad S = \sinh \gamma l, \sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l$$

$S = -.0060556 + j.9912$

$\underline{U = 422.6147 + j10.69768}$

$B \quad 8.04435 \quad j\ 418.8309$

$b_1 = 8.04435$

$b_2 = 418.8309$

$\frac{1}{U} = .002365 + j.0000598716$

$\underline{S = -.0060556 + j.9912}$

$C = \frac{1}{U} S = -.00007419 + j.0023438255$

(c). <u>5th Harmonic.</u>

$r = .147$ ohms per mile.

$x = 3.815$ ohms per mile.

$b = 27.0 \times 10^{-6}$ mhos per mile.

$g = 0$

$z = \frac{x}{\sin \theta_z}$

$\log x = .5814945$

$\log r = 9.1673173-10$

$\log \tan \theta_z = 1.4141772$

$\log \sin \theta_z = 9.9996782-10$

$\log z = .5818163$

$\theta_z = 87^\circ\ 47'\ 40''$

$\log y = 5.4313638-10$

$\theta_y = 90^\circ$

$2 \log v = 6.0131801-10$

$\log v = 8.0065900-10$

$2\ \theta_v = 177^\circ\ 47'\ 40''$

$\theta_v = 88^\circ\ 53'\ 50''$

$v^2 = zy$ | $\log \cos \theta_v = 8.2843386-10$

$2\theta_v = \theta_z + \theta_y$ | $\log \sin \theta_v = 9.9999196-10$

$\log \alpha = 6.2909286-10$

$\log \beta = 8.0065096-10$

$\alpha = v \cos \theta_v$ | $\log \frac{360}{2\pi} = 1.7581230$

$\beta = v \sin \theta_v$ | $\log \beta^\circ = 9.7646326-10$

$\alpha = 0.0001954$

$\beta^\circ = 0.5816$ degree

$2 \log u = 5.1504525$

$u^2 = \frac{z}{y}$ | $\log u = 2.5752262$

$2\theta_u = \theta_z - \theta_y$ | $2\theta_u = -2^\circ\ 12'\ 20''$ $\quad U = 376.3 \underline{/-1^\circ 6' 10''}$

$\frac{1}{u} = \sqrt{\frac{y}{z}}$ | $\theta_u = -1^\circ\ 6'\ 10''$ $\quad \frac{1}{U} = .002659 \underline{/1^\circ 6' 10''}$

$2 \log \frac{1}{u} = 4.8495475-10$

$\log \frac{1}{u} = 7.4247737-10$

$\log \alpha = 6.2909286-10$

$\log l = 2.3979400$

$\log \alpha l = 8.6888686-10$

$\log \beta^\circ l = 2.1625726$

$\alpha l = 0.04885$

$\beta^\circ l = 145.4$ degrees

$\log \cosh \alpha l = 0.0005210$

$\log \sinh \alpha l = 8.6903700-10$

$\log \cos \beta l = 9.9046168-10$ negative

$\log \sin \beta l = 9.7754101-10$

$\log a_1 = \log \cosh\alpha l \cos\beta l = 9.9051578\text{-}10$ negative

$\log a_2 = \log \sinh\alpha l \sin\beta l = 8.4657801\text{-}10$

$a_1 = -.80378$

$a_2 = .029227$

$S = \sinh Vl = \sinh\alpha l \cos\beta l + j \cosh\alpha l \sin\beta l$

$S = -.0393 + j\ .5969$

$\underline{U = 376.23 - j\ 7.2238}$

$B = US = -\ 9.58 + j\ 225.4$

$b_1 = -9.58$

$b_2 = 225.4$

$\frac{1}{U} = .002658 + j\ .000051$

$\underline{S = -.0393\ +\ j\ .5969}$

$C = \frac{1}{U}S = -.0001349 + j\ .001584$

$c_1 = -.0001349$

$c_2 = .001584$

(d) <u>7th Harmonic.</u>

$r = .147$ ohms per mile.

$x = 5.34$ ohms per mile.

$b = 37.8 \times 10^{-6}$ mhos per mile.

$g = 0$

$\log x = .7275413$

$\log r = 9.1673173\text{-}10$

$\log \tan \theta_z = 1.5602240$

$\log \sin \theta_z = 9.9998359\text{-}10$

$\log z = .7277054$

$\theta_z = 88^\circ\ 25'\ 30''$

$\log y = 5.5774918\text{-}10$

$\theta_y = 90^\circ$

$2 \log v = 6.3051972\text{-}10$

$$\log v = 8.1525986-10$$

$$2\,\theta_v = 178^\circ\ 25'\ 30''$$

$$\theta_v = 89^\circ\ 12'\ 45''$$

$$\log \cos \theta_v = 8.1388795-10$$

$$\log \sin \theta_v = 9.9999588-10$$

$$\log \alpha = 6.2914781-10$$

$$\log \beta = 8.1525574-10$$

$$\log \frac{360}{2\pi} = 1.7581230$$

$$\log \beta^\circ = 9.9106804-10$$

$$\alpha = 0.00019565$$

$$\beta^\circ = 0.8141$$

$$\log l = 2.3979400$$

$$\log \alpha l = 8.6894181-10$$

$$\log \beta^\circ l = 2.3086204$$

$$\alpha l = 0.048912$$

$$\beta^\circ l = 203.53 \text{ degrees}$$

$$\log \cosh \alpha l = 0.000521$$

$$\log \sinh \alpha l = 8.690370-10$$

$$\log \cos \beta l = 9.9622878-10 \quad \text{negative}$$

$$\log \sin \beta l = 9.6012803-10 \quad \text{negative}$$

$$\log a_1 = \log \cosh \alpha l \cos \beta l = 9.9628088-10 \quad \text{negative}$$

$$\log a_2 = \log \sinh \alpha l \sin \beta l = 8.2916503-10 \quad \text{negative}$$

$$a_1 = -.91793$$

$$a_2 = -.019573$$

$$\log \sinh \alpha l \cos \beta l = 8.6526578-10$$

$$\log \cosh \alpha l \sin \beta l = 9.6018013-10$$

$$\sinh \alpha l \cos \beta l = -.04494$$

$$\cosh \alpha l \sin \beta l = .39976$$

$S = \sinh Vl = -.0449 + j\ .3997$

$u^2 = \frac{z}{y}$ $\quad 2 \log u = 5.1502136$

$\log u = 2.5751068$

$2\ \theta_u = \theta_z - \theta_y$ $\quad 2\ \theta_u = -1^\circ\ 34'\ 30''$ $\qquad U = 375.9\ \underline{|-0^\circ\ 47'\ 15''}$

$\theta_u = 0^\circ\ 47'\ 15''$ $\qquad \frac{1}{U} = .00266\ \underline{|0^\circ\ 47'\ 15''}$

$B = US = -14.8 + j\ 150.2$

$$2 \log \frac{1}{u} = 4.8497864-10$$

$$\log \frac{1}{u} = 7.4248932-10$$

$C = \frac{1}{U} S = c_1 + j\ c_2 = -.0001316 + j\ .00106164$

(e). 9th Harmonic.

$r = .147$ ohms per mile.

$x = 6.87$ ohms per mile.

$b = 48.6 \times 10^{-6}$ mhos per mile.

$g = 0$

$$\log x = 0.8369567$$

$$\log r = 9.1673173-10$$

$$\log \tan \theta_z = 1.6696394$$

$$\log \sin \theta_z = 9.9999007-10$$

$$\log z = 0.8370560$$

$$\theta_z = 88^\circ\ 46'\ 30''$$

$$\log y = 5.6866363\text{-}10$$

$$\theta_y = 90^\circ$$

$$2 \log v = 6.5236923\text{-}10$$

$$\log v = 8.2618461\text{-}10$$

$$2\,\theta_v = 178^\circ\ 46'\ 30''$$

$$\theta_v = 89^\circ\ 23'\ 15''$$

$$\log \cos \theta_v = 8.0299588\text{-}10$$

$$\log \sin \theta_v = 9.9999751\text{-}10$$

$$\log \alpha = 6.2918049\text{-}10$$

$$\log \beta = 8.2618212\text{-}10$$

$$\log \frac{360}{2\pi} = 1.7581230$$

$$\log \beta^\circ = 0.0199442$$

$$\alpha = 0.00019551$$

$$\beta^\circ = 1.047 \text{ degrees}$$

$$\log l = 2.3979400$$

$$\log \alpha l = 8.6897449\text{-}10$$

$$\log \beta l = 2.4178842$$

$$\alpha l = 0.048949$$

$$\beta^\circ l = 261.75 \text{ degrees}$$

$$\log \cosh \alpha l = 0.0005210$$

$$\log \sinh \alpha l = 8.6903700\text{-}10$$

$$\log \cos \beta l = 9.1668296\text{-}10 \quad \text{negative}$$

$$\log \sin \beta l = 9.9954822\text{-}10 \quad \text{negative}$$

$$\log a_1 = \log \cosh \alpha l \cos \beta l = 9.1673506\text{-}10 \quad \text{negative}$$

$$\log a_2 = \log \sinh \alpha l \sin \beta l = 8.6858522\text{-}10 \quad \text{negative}$$

$a_1 = -.14367$

$a_2 = -.048512$

$u^2 = \frac{z}{y}$ $\quad$ $2 \log u = 5.1504197$

$2\,\theta_u = \theta_z - \theta_y$ $\quad$ $\log u = 2.5752098$ $\quad$ $U = 376 \underline{|-0^\circ\ 36'\ 45''}$

$2\,\theta_u = -1^\circ\ 13'\ 30''$

$\theta_u = -0^\circ\ 36'\ 45''$

$\log \sinh\alpha\,/\cos\beta\,l = 7.8471996-10$ negative

$\log \cosh\alpha\,l \sin\beta\,l = 9.9960032-10$

$\sinh\alpha\,l \cos\beta\,l = -.0070339$

$\cosh\alpha\,l \sin\beta\,l = 0.99084$

$S = \sinh Vl = -.007 + j.9908$ $\quad$ $b_1 = 1.28$

$B = US = 1.28 + j\ 372$ $\quad$ $b_2 = 372$

$2 \log \frac{1}{u} = 4.8495803-10$

$\log \frac{1}{u} = 7.4247901-10$

$\frac{1}{U} = .002659 \underline{|0^\circ\ 36'\ 45''} = .002659 + j.000028$

$C = \frac{1}{U} S = c_1 + jc_2 = -.0000463 + j.00263$

(f). <u>11th Harmonic.</u>

$r = .147$ ohms per mile.

$x = 8.4$ ohms per mile.

$b = 59.4 \times 10^{-6}$ mhos per mile.

$g = 0$

$\log x = 0.9242793$

$$\log r = 9.1673173\text{-}10$$

$$\log \tan \theta_z = 1.7569620$$

$$\log \sin \theta_z = 9.9999335\text{-}10$$

$$\log z = 0.9243458$$

$$\theta_z = 88^\circ\ 59'\ 50''$$

$$\log y = 5.7737864\text{-}10$$

$$\theta_y = 90^\circ$$

$$2 \log v = 6.6981322\text{-}10$$

$$\log v = 8.3490661\text{-}10$$

$$2\ \theta_v = 178^\circ\ 59'\ 50''$$

$$\theta_v = 89^\circ\ 29'\ 55''$$

$$\log \cos \theta_v = 7.9432479\text{-}10$$

$$\log \sin \theta_v = 9.9999833\text{-}10$$

$$\log \alpha = 6.2923140\text{-}10$$

$$\log \beta = 8.3490494\text{-}10$$

$$\log \frac{360}{2\pi} = 1.7581230$$

$$\log \beta^\circ = 0.1071724$$

$$\alpha = 0.00019602$$

$$\beta^\circ = 1.2799 \text{ degrees}$$

$$\log l = 2.3979400$$

$$\log \alpha l = 8.6902540\text{-}10$$

$$\log \beta^\circ l = 2.5051124$$

$$\alpha l = 0.049006$$

$$\beta^\circ l = 319.17 \text{ degrees}$$

$\log \cosh \gamma l = 0.0006210$

$\log \sinh \gamma l = 8.6903700-10$

$\log \cos \beta l = 9.8154854-10$

$\log \sin \beta l = 9.8788748-10$

$\log a_1 = \log \cosh \gamma l \cos \beta l = 9.8160064-10$

$\log a_2 = \log \sinh \gamma l \sin \beta l = 8.5692448-10$ negative

$a_1 = .6546$

$a_2 = -.037089$

$u^2 = \frac{z}{y}$ $\quad 2 \log u = 5.1505594$

$\log u = 2.5752797 \quad U = 376 \angle -0^\circ\ 30'\ 5''$

$2\,\theta_u = -1^\circ\ 0'\ 10'' \quad \frac{1}{U} = .002659 \angle 0^\circ 30'\ 5''$

$\theta_u = -0^\circ\ 30'\ 5''$

$\log \sinh \gamma l \cos \beta l = 8.5058554-10$

$\log \cosh \gamma l \sin \beta l = 9.8793958-10$

$\sinh \gamma l \cos \beta l = 0.032052$

$\cosh \gamma l \sin \beta l \quad 0.75752$

$S = \sinh Vl = .032052 + j.75752$

$B = US = 14.48 + j\ 283.9$

$C = \frac{1}{U} S = .0000675 + j.0020174$

$b_1 = 14.48$

$b_2 = 283.9$

$c_1 = .0000675$

$c_2 = .0020174$

3. TABLE A

Auxiliary Constants Necessary for Transmission Line Calculations.

Harmonic	a_1	a_2	b_1	b_2	c_1	c_2
1	.874	.023	33.3	183	-.000011	.001293
3	-.1393	.043088	8.044	418.83	-.0000419	.0023396
5	-.80378	.029227	-9.58	225.4	-.0001349	.001584
7	-.91793	-.019573	-14.8	150.2	-.0001316	.00106164
9	-.14367	-.048512	1.28	372	-.0000463	.00263
11	.6546	-.03709	14.48	283.9	.0000675	.0020174

4. Sample Calculations for a Sine Wave applied to a 250 Mile Transmission Line.

Given:

Constant load voltage (line to neutral) $E_0 = 127,020$ volts, and load current $I_0 = 197$ amperes. To find E_g and I_g.

(a). Unity Power Factor.

$\underline{/E_0}$ $E = AE_0 + BI_0$

$E_0 = 127,020 + j0$

$I_0 = 197 + j0$

$A = a_1 + ja_2 = .874 + j.023$

From Table A.

$B = b_1 + jb_2 = 33.3 + j183$

$E = (.874 + j.023)(127,020 + j0) + (33.3 + j183)(197 + j0)$

$E = 117,560 + j38930$

$I = AI_0 + CE_0$

$I_0 = 197 + j0$

$A = .874 + j.023$

$E_0 = 127,020 + j0$

$C = c_1 + jc_2 = -.000011 + j.001293$

$I = (.874 + j.023)(197 + j0) + (-.000011 + j.001293)(127,020 + j0)$

$I = 170.4 + j168.5$

(b). Power Factor .85 lagging.

$\underline{/E_0}$ $E_0 = 127,020 + j0$

$I_0 = 197(.85 - j.532) = 167.5 - j105$

$E = (.874 + j.023)(127,020 + j0) + (33.3 + j183)(167.5 - j105)$

$E = 135,600 + j30,130$

$I_0 = 167.5 - j105$

$I = (.874 + j.023)(167.5 - j105) + (-.000011 + j.001293)(127,020 + j0)$

$I = 147.4 + j77.3$

Complete data for values of load current ranging from zero to full load will be found in Table I.

(c). <u>No Load Voltage at Receiving End of Line.</u>

Assuming that the generator voltage is adjusted to give normal rated load voltage at .85 power factor lagging, the no load voltage at the receiving end will be calculated.

$E = AE_0 + BI_0$

$138,700 = E_0\ (.874 + j.023) + BI_0$

For $I_0 = 0$

$$E_0 = \frac{138,700 + j0}{.874 + j.023} = 158,600 - j,4180$$

$E_0 = 158,700$ volts

$E_{0\ max} = 158,700 \times 1.41 = 224,100$ volts.

5. A Non-harmonic Wave (as it originates at generator.)

The following wave whose effective value at the generator is the same as that of the sine wave previously considered, namely 138,700 volts will now be applied to the line.

$$e_{gen} = 195{,}000 \sin \theta$$
$$+15{,}000 \sin (3\theta - 17.2^\circ)$$
$$+8{,}000 \sin (5\theta - 160^\circ)$$
$$+6{,}000 \sin (7\theta - 10^\circ)$$
$$+3{,}000 \sin (9\theta \quad 145^\circ)$$
$$+700 \sin (11\theta - 205^\circ)$$

6. Calculations for Voltage Wave Equation at Receiving End on Open Circuit.

$E = AE_0 + BI_0$ Values of A from Table A.

For $I_0 = 0$ $E_0 = \dfrac{E}{A}$

(a). Fundamental.

$\angle E_1$

$$E_0 = \frac{195{,}000}{.874 \quad j.023}$$

$$E_0 = 223{,}000 - j5{,}880 = 223{,}500 \ \angle -1.5^\circ$$

(b). 3rd Harmonic.

$\angle E_3$

$$E_0 = \frac{15{,}000(-.1393 - j.0431)}{.0212}$$

$$E_0 = 103{,}400 \ \angle -162.8^\circ$$

(c). 5th Harmonic.

$$E_0 = \frac{8{,}000(-.8038 - j.0292)}{.6468}$$

$\angle E_5$

$$E_0 = -9{,}950 - j\,362 = 9{,}950 \ \angle -177.9^\circ$$

(d). 7th Harmonic.

$\angle E_7$

$$E_0 = \frac{6{,}000(-.91793 \quad j.01957)}{.84036}$$

$$E_0 = -6{,}565 \quad j\,117.3 = 6{,}565 \ \angle 179^\circ$$

(e). <u>9th Harmonic.</u>

$$E_0 = \frac{3,000(-.14367 + j.048512)}{.02295}$$

$$E_0 = -18,750 + j\ 6,340 = 19,800 \ \underline{|161.4^\circ}$$

(f). <u>11th Harmonic.</u>

$$E_0 = \frac{700(.6546 \quad j.03709)}{.42937}$$

$$E_0 = 1,070 + j\ 60.3 = 1,070 \ \underline{|3.23^\circ}$$

The above values of E_0 are maximum values.

7. Equation of the Voltage Wave at Receiving End of Line on Open Circuit.

From the data obtained in (6) and the equation of the original wave from (5) the equation of the E.M.F. wave at the load is readily determined to be:

$$\begin{aligned}
e_0 = & \ 223,500 \sin (\theta - 1.5^\circ) \\
& + 103,400 \sin (3\theta - 180^\circ) \\
& + 9,950 \sin (5\theta - 337.9^\circ) \\
& + 6,565 \sin (7\theta + 169^\circ) \\
& + 19,800 \sin (9\theta + 306.4^\circ) \\
& + 1,070 \sin (11\theta - 201.77^\circ)
\end{aligned}$$

The effective value of this wave is 176,500 volts.

8. Harmonic Load Current and Load Voltage Calculations. (r_0 and L_0 same as for sine wave applied.)

$E_0 = I_0 Z_0$ for any harmonic.

$E = AE_0 + BI_0$

$E = AI_oZ_o + BI_o$

$I_o(AZ_o + B) = E$

$I_o = \frac{E}{AZ_o + B}$

All values of A and B taken from Table A.

All values of E taken from equation in Part (5).

The above equations will now be used to determine all harmonics of load current and load voltage and all phase relations.

(a). Fundamental.

$\underline{/E_1}$ $I_o = \frac{195,000 + j0}{(.874 + j.023)(550 + j338) + (33.3 + j183)}$

$I_o = 198.5 - j192.5 = 276 \underline{/-44.3^\circ}$

$Z_o = 646 \underline{/32^\circ}$

$i_o = 276 \sin(\theta - 44.3^\circ)$

$e_o = 178,500 \sin(\theta - 12.3^\circ)$

(b) 3rd Harmonic.

$\underline{/E_3}$ $Z_o = 550 + j1,014 = 1,150 \underline{/61.4^\circ}$

$I_o = \frac{15,000 + j0}{(-.1393 + j.042)(550 + j1,014) + (8.47 + j422)}$

$I_o = -15.85 - j\,43.3 = 46.3 \underline{/-110.1^\circ}$

$i_o = 46.3 \sin(3\theta - 127.3^\circ)$

$e_o = 52,200 \sin(3\theta - 65.9^\circ)$ $E_{3\,max} = 52,200 \underline{/-48.7^\circ}$

(c) 5th Harmonic.

$\underline{/E_5}$ $Z_o = 550 + j\,1,690 = 1,775 \underline{/71.9^\circ}$

$I_o = \frac{8,000 + j0}{(-.803 + j.029)(550 + j1,690) + (-9.58 + j\,225.4)}$

$I_0 = -2.65 + j\ 5.92 = 6.48\ \underline{/114.1^\circ}$

$i_0 = 6.48 \sin (5\ \theta - 45.9^\circ)$

$e_0 = 11,500 \sin (5\ \theta + 26^\circ) \qquad E_{0\ max} = 11,500\ \underline{/186^\circ}$

(d). <u>7th Harmonic.</u>

$\underline{/E_7}$ $\qquad Z_0 = 550 + j\ 2,375 = 2,435\ \underline{/76.9^\circ}$

$$I_0 = \frac{6,000 + j0}{(-.918 - j.0196)(550 + j\ 2,375) + (-14.8 + j\ 150.2)}$$

$I_0 = -.645 + j\ 2.78 = 2.85\ \underline{/103.1^\circ}$

$i_0 = 2.85 \sin (7\ \theta + 93.1^\circ)$

$e_0 = 6,950 \sin (7\ \theta + 170^\circ) \qquad E_{0\ max} = 6,950\ \underline{/180^\circ}$

(e) <u>9th Harmonic.</u>

$\underline{/E_9}$ $\qquad Z_0 = 550 + j\ 3,050 = 3,100\ \underline{/79.8^\circ}$

$$I_0 = \frac{3,000 + j0}{(-.1437 - j.0485)(550 + j\ 3,050) + (1.28 + j\ 372)}$$

$I_0 = .934 + j\ 6.28 = 6.34\ \underline{/81.5^\circ}$

$i_0 = 6.34 \sin (9\ \theta + 226.5^\circ)$

$e_0 = 19,700 \sin (9\ \theta + 306.3^\circ) \qquad E_{0\ max} = 19,700\ \underline{/161.3^\circ}$

(f). <u>11th Harmonic.</u>

$\underline{/E_{11}}$ $\qquad Z_0 = 550 + j\ 3,730 = 3,740\ \underline{/81.5^\circ}$

$$I_0 = \frac{700 + j0}{(.6546 - j.0371)(550 + j\ 3,730) + (14.48 + j\ 283.9)}$$

$I_0 = .0473 - j.2495 = .2545\ \underline{/-79.3^\circ}$

$i_0 = .2545 \sin (11\ \theta - 234.3^\circ)$

$e_0 = 953 \sin (11\ \theta - 202.8^\circ) \qquad E_{0\ max} = 953\ \underline{/2.2^\circ}$

9. Equations for Voltage and Current at Receiving End under Full Load. ($I_o = 198$ amperes)

$$
\begin{aligned}
e_o = {} & 178{,}500 \sin(\theta - 12.3^\circ) \\
& + 52{,}200 \sin(3\theta - 65.9^\circ) \\
& + 11{,}500 \sin(5\theta + 26^\circ) \\
& + 6{,}950 \sin(7\theta + 170^\circ) \\
& + 19{,}700 \sin(9\theta + 306.3^\circ) \\
& + 953 \sin(11\theta - 202.8^\circ)
\end{aligned}
$$

$$E_o \text{ eff.} = \frac{\sqrt{E_{1m}^2 + E_{3m}^2 + \cdots}}{1.41} = 132{,}600 \text{ volts.}$$

$$
\begin{aligned}
i_o = {} & 276 \sin(\theta - 44.3^\circ) \\
& + 46.3 \sin(3\theta - 127.3^\circ) \\
& + 6.48 \sin(5\theta - 45.3^\circ) \\
& + 2.85 \sin(7\theta + 93.1^\circ) \\
& + 6.34 \sin(9\theta + 226.5^\circ) \\
& + .254 \sin(11\theta - 284.3^\circ)
\end{aligned}
$$

$$I_o \text{ eff.} = \frac{\sqrt{I_{1m}^2 + I_{3m}^2 + \cdots}}{1.41} = 198 \text{ amperes}$$

The effective voltage at the load is:

$$E = .707\sqrt{178{,}500^2 + 52{,}200^2 + 11{,}500^2 + 6{,}950^2 + 19{,}700^2 + 953^2}$$

$$= 132{,}600 \text{ volts.}$$

10. Generator Current Calculations for Full Load at End of Line.

Having obtained the maximum values of all harmonics of current and voltage at the load and the angles showing their phase relation in (9) we can now proceed to obtain the current at the generator. Since these data are to be used for Table II to obtain the power contributed by each of the harmonics we will now deal with <u>effective</u> values.

All values of E_0 and I_0 used in the following calculations are taken from part (9) and reduced to effective values.

The complex numbers A and C are taken directly from Table A.

$I = AI_0 + CE_0$

(a). Fundamental.

$I = (.874 + j.023)(140.5 - j130) + (-.000011 + j.001292)(123,200 - j26,900$

$I = 159.18 + j43.53 = |165|$

(b). 3rd Harmonic.

$I = (-.1393 + j.043088)(-11.2 - j30.6) + (-.0000756 + j.0023396)$
$(24,400 - j27,700)$

$I = 65.84 + j\ 62.88 = |91|$

(c) 5th Harmonic.

$I = (-.80378 + j.029227)(-1.87 + j\ 4.18) + (-.0001349 + j.001584)$
$(-8,090 + j\ 850)$

$I = 1.123 - j\ 16.349 = |16.4|$

(d) 7th Harmonic.

$I = (-.91793 - j.019573)(-.456 + j\ 1.965) + (-.0001316 + j.00106164)$
$(-4,920 + j0)$

$I = 1.0965 - j\ 7.016 = |7.1|$

(e) 9th Harmonic.

$I = (-.14367 - j.048512)(.66 + j\ 4.43) + (-.0000463 + j.00263)$
$(-13,200 + j4,470)$

$I = -11.0 - j35.67 = |37.3|$

(f) 11th Harmonic.

$I = (.6546 - j.03709)(.0335 - j.1765) + (.0000675 + j.0020174)$
$(675 + j\ 25.9)$

$I = -.00623 + j\ 1.353 = |1.36|$

The foregoing calculations were based upon an effective voltage at the generator of 138,700 volts.

$$I = \sqrt{165^2 + 91^2 + 16.4^2 + 7.1^2 + 37.3^2 + 1.36^2}$$

$I = 192.5$ amperes.

11. No Load Generator Current and Power Calculations.

$E = AE_0 + BI_0 \qquad I_0 = 0$

$I = AI_0 + CE_0$

$$I = \frac{EC}{A}$$

(a) Fundamental.

$$I = \frac{(110{,}000 + j0)(-.000011 + j.001292)}{.874 + j.023}$$

$\angle E_1$

$I = 2.48 + j\,147 \qquad I = 147$ amps.

$E = 110{,}000 + j0$

$P = 273{,}000$ watts

$= 273$ K.W.

(b) 3rd Harmonic.

$$I = \frac{8{,}400(-.0000756 + j.002339)}{-.1393 + j.043}$$

$\angle E_3$

$I = 44 - j127.7 \qquad I = 135$ amps.

$E = 8{,}400 + j0$

$P = 370{,}000$ watts

$= 370$ K.W.

(c) 5th Harmonic.

$$I = \frac{4{,}520(-.0001349 + j.001584)}{-.8038 + j.02923}$$

$\angle E_5$

$I = 1.075 - j\,8.85 \qquad I = 8.92$ amps.

$E = 4{,}520 + j0$

$P = 4{,}870$ Watts

(d) 7th Harmonic.

$$I = \frac{3,390(-.0001316 + j.00106)}{-.918 - j.0196}$$

$I = .4 - j\,3.92$ $\qquad$ $I = 3.94$ amps.

$E = 3,390 + j0$

$P = 1,355$ watts

$= 1.355$ K.W.

(e) 9th Harmonic.

$$I = \frac{1,696(-.0000463 + j.00263)}{-.144 - j.0485}$$

$I = -8.9 - j\,28.1$ $\qquad$ $I = 29.43$ amps.

$E = 1,696 + j0$

$P = -15,100$ watts

$= -15.1$ K.W.

(f) 11th Harmonic.

$$I = \frac{396(.0000675 + j.00202)}{.6546 - j.0371}$$

$I = -.0285 + j\,1.223$ $\qquad$ $I = 1.22$ amps.

$E = 396 + j0$

$P = -11.3$ watts

$= -.0113$ K.W.

Total power at no load $= 273 + 370 + 4.87 + 1.35 + 15.1 + .0113$

$= 634.11$ K.W.

$$I = \sqrt{147^2 + 135^2 + 8.92^2 + 3.94^2 + 29.43^2 + 1.22^2}$$

$= 202$ amperes (effective value of current at the generator at no load)

$$E = \sqrt{110,000^2 + 8,400^2 + 4,620^2 + 3,390^2 + 1,695^2 + 396^2}$$

$=111,000$ volts (effective value of voltage at the generator at no load)

12. Load Power Calculations. ($I_{0\,eff.} = 198$ amperes)

Let $e = E_1 \sin(\theta + \phi_1) + E_3 \sin(3\theta + \phi_3)$

$$+ \cdots + E_n \sin(n\theta + \phi_n)$$

and $i = I_1 \sin(\theta + \delta_1) + I_3 \sin(3\theta + \delta_3)$

$$+ \cdots + I_n \sin(n\theta + \delta_n)$$

Then $P = \frac{E_1 I_1}{2} \cos(\phi_1 - \delta_1) + \frac{E_3 I_3}{2} \cos(\phi_3 - \delta_3)$

$$+ \cdots + \frac{E_n I_n}{2} \cos(\phi_n - \delta_n)$$

where δ and ϕ may be either positive or negative.

Maximum values of E_0 and I_0 are taken from Part (9).

$$P = \frac{178,500 \times 276}{2} \cos 32^\circ + \frac{52,200 \times 46.3}{2} \cos 61.4^\circ$$

$$+ \frac{11,500 \times 6.48}{2} \cos 71.9^\circ + \frac{6,950 \times 2.85}{2} \cos 76.9^\circ$$

$$+ \frac{19,700 \times 6.34}{2} \cos 79.8^\circ + \frac{953 \times .254}{2} \cos 81.5^\circ$$

$P = 20,900 + 577 + 11.6 + 2.24 + 11.05 + .0177 = 21,502$ K.W.

13. Determination of Reactive Power.

The reading of theR.K.V.A.H. meter divided by the time in hours will be

$$P_r = \frac{E_1 I_1}{2} \sin(\phi_1 - \delta_1) + \frac{E_3 I_3}{2} \sin(\phi_3 - \delta_3)$$

$$+ \cdot \cdot \cdot \cdot \cdot + \frac{E_n I_n}{2} \sin(\phi_n - \delta_n)$$

This is the so called reactive power as would be determined from a R.K.V.A.H. meter reading after dividing by the time. It is <u>not</u> the reactive power according to the A.I.E.E. definition. Maximum values of E_0 and I_0 and phase angles will be taken from Part (9).

$$P_r = \frac{178.5 \times 276 \times .5299}{2} + \frac{52.2 \times 46.3 \times .878}{2} + \frac{6.48 \times 11.5 \times .9905}{2}$$

$$+ \frac{6.95 \times 2.85 \times 9.74}{2} + \frac{19.7 \times 6.34 \times .9842}{2} + \frac{.953 \times .254 \times .989}{2}$$

$P_r = 13{,}050 + 1{,}060 + 35.5 + 9.65 + 62.2 + .12 = 14{,}217$ K.V.A.

14. Power Factor as Measured by the Power Company.

Tan θ is found on the assumption of sine waves.

Hence $\quad \text{Tan } \theta = \frac{\text{R.K.V.A.}}{\text{K.W.}} = \frac{14{,}217}{21{,}502} = .661$

From a table of trigonometric functions

Cos θ = .7501.

This is the power factor measured at the load which the power company would find from their measurements and upon which their rate structures would be based.

15. Power Factor according to A.I.E.E. Definition.

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}} = \frac{21{,}502}{26{,}250} = .82$$

where 21,502 K.W. is the power found in (12) and 26,250 K.V.A. is the apparent power found by multiplying the r.m.s. value of the load voltage by the r.m.s. value of the load current and dividing by one thousand.

VIII SUMMARY

From the results of this research a number of facts have been shown to exist.

1. On a typical 250 mile, 220,000 volt transmission line built to present standards of good practise it has been shown:

   A. That an e.m.f. wave containing a 7% third harmonic will contain a 53% third harmonic upon reaching the end of the line under no load conditions.

   B. That even under full load conditions the 3rd harmonic content of the wave at the end of the line is 29% of the fundamental.

   C. The power factor at the generator at full load (according to the definition of power factor in the A.I.E.E. standards) is .848 for a wave containing harmonics compared with .96 for a purely sinusoidal wave.

   D. <u>The power factor at the load (according to A.I.E.E. definition) is .82, but the power factor as determined by the power companies measuring instruments is .75.</u>

2. Methods of Metering.

   A. The present method of using a phase shifting transformer to obtain the ninety degree relation necessary for measurement of reactive power is totally unreliable when used on waves containing harmonics.

B. Watt-hour meters are subject to error when used on non-sinusoidal waves.

3. An e.m.f. wave containing harmonics was applied to an induction motor and the following facts were determined:

   A. An nth harmonic in the e.m.f. wave will produce a harmonic of current in the rotor circuit whose frequency is f_r $f(n - 1)$ s

      where f_r frequency of rotor current

      f frequency of applied e.m.f.

      s slip expressed in cycles per second and considered positive when the motor runs at sub-synchronous speed.

   B. A harmonic of opposite phase sequence caused the motor to vibrate and make a periodic rasping sound.

   C. The rotor current at no load for e.m.f. containing a 33% 2nd harmonic was twenty times the value measured when a sine wave was used.

   D The wattmeter reading for the second harmonic was about three times the no load reading for a sine wave.

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